INVESTIGATING THE ROLE OF HISTORY OF MATHEMATICAL CONCEPTS IN LEARNING MATHEMATICS IN UPPER SECONDARY SCHOOL LEVEL IN MALAWI

M.Ed. (CURRICULUM AND TEACHING STUDIES – SCIENCE AND MATHEMATICS EDUCATION) THESIS

COSMAS JOHN KATHUMBA

UNIVERSITY OF MALAWI

CHANCELLOR COLLEGE

MAY, 2017

INVESTIGATING THE ROLE OF HISTORY OF MATHEMATICAL CONCEPTS IN LEARNING MATHEMATICS IN UPPER SECONDARY SCHOOL LEVEL IN MALAWI

M.Ed. (CURRICULUM AND TEACHING STUDIES – SCIENCE AND MATHEMATICS EDUCATION) THESIS

By

COSMAS JOHN KATHUMBA

BSc (Education)-Mzuzu University

Submitted to the Department of Curriculum and Teaching Studies, School of Education, in partial fulfillment of the requirements for the degree of Master of Education in Curriculum and Teaching Studies (Science and Mathematics Education)

University of Malawi

Chancellor College

May, 2017

DECLARATION

I the undersigned hereby declare that this thesis/dissertation is my own original work which has not been submitted to any other institution for similar purposes. Where other people's work has been used acknowledgements have been made.

_____

Full Legal Name

_____

Signature

_____

Date

CERTIFICATE OF APPROVAL

The undersigned certify that this thesis represents the student's own work and effort and has been submitted with our approval.

Signature: ____  ___ Date: _____

Reidar Mosvold, PhD (Professor)

Main Supervisor

Signature: _____Date:_____

Mercy Kazima Kishindo, PhD (Associate Professor)

Co-Supervisor

Signature: _____Date _____

Amos Chauma, PhD (Lecturer)

Head of Department

DEDICATION

I dedicate this thesis to my late father, Mr. John Kathumba for his encouragement before he died 19 years ago. May his soul rest in eternal peace.

ACKNOWLEDGEMENTS

I would like to thank the Almighty God for the wisdom and all the support I got to complete this study.

I would like to thank my supervisor, Professor Reidar Mosvold of the University of Stavanger, Norway, for the unlimited support he rendered to me. Without his supervisory support, I would not have completed my study in good time. I also acknowledge the supervisory support of Associate Professor Mercy Kazima Kishindo in writing this Masters thesis.

I also thank the Malawi government for the financial support they provided to me by paying the tuition fees through the Malawi Government Scholarship Fund.

I am also thankful to the school head teacher, the teacher and the learners for their support during the data generation stage of this study.

Finally, I would like to thank my family, my wife Lucia Lemani for her support and encouragement throughout the study, my daughters Favour and Rejoice, my uncle Mr. Thomas Mose and his family, my brothers and sisters for their encouragements. The list cannot be complete without acknowledging my cousin, Edward Mose, who provided me with his laptop computer that made the writing process easier. May Almighty God bless them all.

ABSTRACT

The purpose of this study was to explore the role of using history of mathematical concepts in teaching and learning mathematics in upper secondary school level in Malawi. The study used a qualitative case study approach in which 50 Form Three learners and their Mathematics teacher from one of the schools in Zomba urban were conveniently sampled to participate. Data were generated through classroom observation, face to face interviews and questionnaires. Data were analysed qualitatively by coding, transcribing and classifying the data into themes guided by the research sub-questions. The findings of this study have revealed that using history of mathematical concepts in teaching and learning mathematics increase motivation and learners' learning as it: (a) enhance learners' mathematical concept understanding; (b) enhance their creativity in mathematics; (c) enable learners to engage in different learning strategies and (d) enable learners to easily recall some concepts and make references to various historical information. The findings have also indicated that learners develop positive attitude towards mathematics. The study concluded that there are a number of benefits learners and the teacher would gain if historical information of some mathematical concepts is used in the teaching and learning mathematics. The implication is that if history is not used in teaching and learning mathematics, many learners will still lack motivation that will affect how they learn and understand many mathematical concepts.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF APPENDICES

CHAPTER ONE

INTRODUCTION

1.0 Chapter Overview

This chapter presents background of the study, relevance of the thesis, statement of the problem, the purpose of the study, objectives of the study, the main research question and sub-questions, significance of the study, definitions of terms and structure of the thesis.

1.1 Background of the study

Mathematics is considered as one of the oldest fields of study in the history of mankind. It contains some of the most vital components of human thought that have existed for centuries. Mathematics is also described as a field of study that sharpens the human mind, develops their logical thinking, and enhances their reasoning ability (Siu, 2000). It has also been noted that some science and technological developments in the world are enhanced by the use of mathematical ideas (Mapotse & Gumbo, 2015). These developments are possible because different people are able to use other mathematical ideas that were developed many centuries ago to develop new ideas in mathematics and science. Basing on the importance of mathematics in everyday activities and its application in various disciplines, mathematical ideas and concepts are taught at different levels of an individual's development, be it at kindergarten, primary, secondary or tertiary.

The researcher is a trained secondary school mathematics teacher and was exposed to many mathematical concepts from different branches of mathematics. However, he was not taught explicitly how these concepts were developed and why they are selected to be taught at different levels of education until he was enrolled into the Master of Education programme. During the First Year of the Master's course in the history and pedagogy of mathematics, the researcher came into the realisation of how some of the mathematical concepts were developed and the way the secondary school mathematics in Malawi and other countries are presented in the curriculum and the textbooks.

The secondary school mathematics curriculum for Malawi outlines how different mathematical concepts are taught at different levels. These mathematical concepts are presented in such a way that would enable learners to develop their logical thinking and enhance their reasoning ability from lower level to upper level and be able to use previous concepts on new concepts learnt at another level (Malawi Institute of Education, 2013). Learners at different educational levels learn various mathematical concepts in Algebra, Arithmetic, and Geometry. However, most of the primary and secondary school learners even those at tertiary level have insufficient knowledge about how these concepts came into existence, where they were developed and why some were adopted and others were not. This implies that learners at secondary school level are also not much exposed to the history of mathematical concepts they are learning since the Malawian education system does not teach the history of mathematics to both pre-service and in-service teachers.

When textbooks that are used at both lower and upper secondary school levels in Malawi were reviewed, it was discovered that few textbooks add some elements of history of some mathematical concepts such as names of some ancient mathematicians and brief biographies. For instance, a Form Four (last class in upper level) mathematics textbook contains the brief biography and achievements of Carl Gauss and the century he lived. The intention of presenting such information in the textbooks to most teachers and learners is not clear until someone has knowledge of history of mathematics. For instance, someone who has the knowledge of history of mathematical concepts considers such information as a tool to attract learners' attention, motivate them to develop their own critical thinking skills in mathematics (Fauvel & van Maanen, 2002).

In addition, Malawi's secondary school mathematics curriculum emphasises on the need for learners to develop skills such as reasoning, critical thinking, and problem solving through the learning and application of mathematics (Malawi Institute of Education, 2013). So, using history to learn mathematics would bring in what Fried (2007, p. 207) referred to as the "clash of commitment" since learners are supposed to understand the mathematics content. The curriculum also points out that when mathematical ideas are discovered and insights gained, learners should be encouraged to pursue mathematics beyond the classroom walls. What the curriculum wants learners to achieve in mathematics is relevant to the benefits of using history of mathematical concepts in mathematics education. For instance, learners are supposed to develop different skills in mathematics and this could be achieved if learners are motivated towards learning the subject (e.g. Jankvist, 2009). Motivation is considered as one of the factors that lets

learners enjoy learning mathematics but one would ask "what motivates learners?" Fauvel and van Maanen (2002) suggest that using history of mathematical concepts in the mathematics classroom could motivate learners because they could see the relevance of learning mathematics basing on the past trends and the applications of mathematical ideas in various fields. On the other hand, other scholars argue that integrating the history of mathematical concepts into classroom activities does not automatically change learners' motivation but it can lead learners to view mathematics from another perspective that would enable them to think that mathematics is a human work (e.g. Avital, 1995; Fauvel, 1991).

Furthermore, learners do not just learn mathematical concepts on their own, or by using a single learning strategy, but teachers play a crucial role in this situation. It is believed that learners respond positively to learning mathematics if their teachers encourage the use of different learning strategies (Kaphesi, 2015). So, one of the strategies teachers might use in classroom could be, to include some information that would enhance learners' understanding of mathematical concepts such as the history of the concept they are learning. Burns (2010) states that learning mathematics is influenced by the conception of the mathematics itself and this does not depend only on the mathematical content knowledge of the teacher but also on the meaningful mathematical content base (knowing the reasons and ways/means of teaching and learning mathematics). So, the study of history of mathematical concepts could enable learners to understand the reasons and ways of mathematics (Jankvist, 2009). For instance, it has been found out that those learners who have undergone the lessons that incorporated the history of mathematical

concepts performed significantly better than those who did not (Horton & Panasuk, 2011). The history of mathematical concepts might help learners appreciate how mathematical ideas were developed in the past, hence aiding their learning process.

1.2 Relevance of the thesis

The idea of conducting this research about the role of history of mathematical concepts in mathematics learning was motivated by the International Commission on Mathematical Instruction (ICMI) study, which was published in a book edited by Fauvel and van Maanen (2002). The book summarises many issues regarding the use of history of mathematics in either mathematics curriculum, textbooks or in the classroom for several countries in Europe, America and Asia. It has been observed that history of mathematics in these regions was incorporated at different times. Some of them started using it explicitly in the early 1970s, for instance, in Denmark (Fauvel & van Maanen, 2002). History of mathematics as presented in the ICMI study, was first incorporated into the tertiary level mathematics curriculum where teachers were trained to acquire the historical knowledge of some mathematical concepts and later, other levels of education such as secondary and primary were considered.

There are several areas in the mathematics curriculum where history of mathematical concepts might be incorporated, since there are many branches of mathematics. For instance, Coordinate Geometry is one of the mathematics topics that was discovered around 17th century and is still being taught in schools, therefore, its history can be incorporated into the classroom. The history that might be incorporated into the

classroom include: how the topic was developed, the century the topic was discovered, mathematicians that were involved in developing the concepts in the topic. The background on the topic if incorporated into the mathematics classroom is believed to enable both teachers and learners in understanding the concepts within the topic much better (Fauvel & van Maanen, 2002). It was also observed that the topic and its concepts were gradually developed and the views of other mathematicians were accommodated to help in improving the concepts so that current generation is able to understand the ideas to be imparted to them (Merzbach & Boyer, 2011).

Therefore, this study aimed at finding some benefits of incorporating the history of some mathematical concepts from the chosen mathematics topic on learners' understanding and reasoning skills as well as how history might be implemented in the mathematics classroom. The study also sought to understand or explore some of the challenges of incorporating history of mathematical concepts in mathematics learning.

1.3 Statement of the Problem

The meaningful teaching and learning of mathematics in schools is supposed to support the development of knowledge and understanding among teachers as well as learners on the origin of the mathematical concepts (Bütüner, 2015). Since Malawian education system does not teach history of mathematics to pre–service or in–service teachers, most secondary school teachers have insufficient knowledge of history of mathematics in general. The insufficient knowledge teachers might have on the historical development of some mathematical concepts could also result in insufficient knowledge of history of

mathematical concepts in learners as well and affect their teaching and learning process. History of some mathematical concepts used in mathematics teaching and learning is believed to benefit both teachers and learners (Barnett, Lodder & Pengelley, 2013; Goktepe & Ozdemir, 2013; Horton & Panasuk, 2011) in terms of development of knowledge on different mathematical concepts and their motivation.

Teachers and educators have tried to find ways of motivating learners to develop an interest towards mathematics so that they improve their understanding of concepts. Different factors could influence learners' motivation and attitude towards mathematics. The researcher hopes that integrating history of some mathematical concepts in learning mathematics could play a vital role in developing learners' motivation that leads to understanding of concepts and learners' attitude towards mathematics and how learners learn mathematics.

1.4 Purpose of Study

The study sought to explore the possible benefits of using history of mathematical concepts in learning mathematics in classroom, in addition to exploring learners' attitude towards mathematics after the learners are exposed to history and understanding of mathematical concepts. The study also sought to explore how the history of mathematical concepts might be incorporated and the challenges learners could face if it is incorporated in the mathematics classroom in upper secondary school level in Malawi.

1.5 Objectives of Study

The following were the study objectives:

1. To explore possible benefits of using history of mathematical concepts to support the understanding of concepts in mathematics among learners
2. To find out if using history of mathematical concepts in classroom had the potential to influence learners' attitude towards mathematics in general
3. To find out when and how history of mathematical concepts could be incorporated in mathematics classrooms
4. To investigate challenges of implementing the history of mathematical concepts in mathematics classroom.

1.6 Main research question

How can the use of history of mathematical concepts enhance the process of learning mathematics in upper secondary school level in Malawi?

1.6.1 Research sub-questions

The following sub-questions were used to answer the main research question.

1. What are some benefits of using history of mathematical concepts in developing the understanding of mathematical concepts in learners?
2. What are some effects of using history of mathematical concepts in classroom on learners' attitude towards mathematics?

3. When and how can history of mathematical concepts be incorporated into the mathematics classroom?
4. What are some challenges of implementing the history of mathematical concepts in mathematics classroom?

1.7 Significance of study

It is important for Malawi, as a developing country, to allow its citizens to know and understand how the relevant mathematical concepts in schools, colleges and universities were developed, selected and presented at different levels of education. For instance, Calculus is not introduced into the normal mathematics curriculum for primary and secondary schools but in the Additional Mathematics for secondary school. Mathematical concepts are taught to learners basing on their complexity and that knowledge is very important to curriculum developers and mathematics teachers. Therefore, the findings of this study can inform a number of stakeholders presented below in different ways.

1.7.1 The researcher and other mathematics teachers

It is believed that the researcher and other mathematics teachers would gain insight about the significance of considering and incorporating history of mathematical concepts from the chosen mathematics topic and other topics in mathematics in the mathematics classroom. As such, teachers might be interested to include some historical information of mathematical concepts that would facilitate the teaching and learning process. The researcher and other mathematics teachers would also acquire knowledge about the challenges of integrating the history of mathematical concepts in the classroom.

1.7.2 Mathematics Teacher Educators

It is believed that the findings of this study can inform mathematics teacher educators to train prospective mathematics teachers on the importance of using history of some mathematical concepts from the chosen mathematics topic and other topics in mathematics.

1.7.3 Mathematics education researchers

The findings of this study may also be used as a basis for further studies within the secondary school level but on different participants.

1.7.4 Curriculum developers

Knowledge about the possible benefits and challenges of using history of mathematical concepts in mathematics education would enable curriculum developers to incorporate relevant topics and themes of the history of mathematics into the curriculum.

1.8 Definition of terms

Attitudes: "Manners of acting, feeling, or thinking that show one's disposition or opinion" (Philipp, 2007, p. 259). Philipp further points out that attitudes change over time, and they change more quickly than beliefs. In addition, attitudes may involve positive or negative feelings about a phenomenon and they are felt with less intensity. Attitudes also affect a person's mentality.

Beliefs: "Psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). Philipp further suggests that beliefs are more cognitive than attitudes as such they are harder to change than attitudes. Beliefs are also considered as lenses that affect one's view of some aspect of the world or as dispositions toward action.

History: "The way that a particular subject has developed or changed throughout its existence" (MacMillan English Dictionary, 2002, p. 679)

Learning Strategies: Refer to learners' self-generated thought, feelings and actions that are systematically oriented towards the attainment of their goals (Kaphesi, 2015). Learning strategies are fundamental to the process of acquiring knowledge as well as the skills of reading, questioning, reasoning, problem solving and memorisation.

Perception: "The ability to understand and make good judgement about something" (MacMillan English Dictionary, 2002, p. 1051).

1.9 Structure of the thesis

The thesis reports the study that investigated the role of history of mathematical concepts in learning mathematics in upper secondary school level. The thesis has the following five chapters: Introduction, Review of related literature and research, research design and methodology, results and discussion of findings and conclusion, implications and recommendations.

The introduction has outlined the study by providing and discussing issues relating to the background of the study, statement of the problem, the purpose of the study, objectives of the study, the research questions and the significance of the study. The chapter has also looked at the definition of key terms as used in the study. The chapter on review of related literature and research presents the issues related to the role of using history of mathematical concepts in mathematics learning and the theoretical framework of the study. The research design and methodology chapter discusses the research design and introduces the participants and sampling techniques, research instruments, data analysis and interpretation and the ethical considerations of the study. In the chapter on results and discussion of findings, the findings are presented in thematic form basing on the research sub-questions. Data generated from the three instruments were analysed and the findings are presented according to their commonality. The four research sub-questions are used as the headings on each sub-section. Finally, the conclusion, implications and recommendations chapter presents the conclusion of the findings of the study, the implications of the findings for teaching and learning mathematics and the recommendations for the implementations of the findings, and for further studies.

CHAPTER TWO

REVIEW OF RELATED LITERATURE AND RESEARCH

2.1 Introduction

The study used Coordinate Geometry as a mathematical topic in focus, and the history of the topic will be outlined along with a presentation of the birth of present-day Coordinate Geometry. The chapter also discusses the reasons of incorporating history of mathematical concepts in classrooms – with a focus on the benefits of integrating history of mathematical concepts in mathematics classroom. History of mathematical concepts and learners' attitude towards mathematics and the challenges of incorporating history of mathematics in the classroom have been addressed. Two categories of arguments of using history of mathematical concepts in mathematics education – history as a tool and history as a goal – will be discussed. In addition, some approaches to implementing history of mathematical concepts will be discussed and these are: the illumination approaches, the module approaches and the history-based approaches. The constructivism theory has been discussed too.

2.2 History of Cartesian/coordinate geometry

Cartesian/coordinate geometry is also referred to as Analytic Geometry and was discovered in the 17th century by two French mathematicians called Renè Descartes (1596–1650) and Pierre de Fermat (1601–1665) (Katz, 2009; Stillwell, 2010). These mathematicians independently discovered it – Fermat in 1629 and Descartes in 1637. The work of Fermat was more systematic than that of Descartes, but Descartes published his work first (Boyer, 1968; Katz, 2009; Stillwell, 2010). Descartes' work is distinguished by its use of a coordinate axis system to aid in algebraic derivations, and this laid the foundation for analytic geometry. Today, we call this representation of a locus of points on a system of reference axes Cartesian in his honour (Merzbach & Boyer, 2011).

Merzbach and Boyer (2011) report that the work of Descartes was far too often described simply as the application of algebra to geometry. For instance, Descartes began his work with geometry and built his axes into his geometry. In addition, Descartes ignored the negative roots, as he considered these to be false roots (e.g., Stillwell, 2010). He did not lay down a coordinate frame to locate points as a surveyor or a geographer might do, nor were his coordinates thought of as number pairs (Merzbach & Boyer, 2011). Whereas Descartes began his work with geometry and built his axes into his geometry, Fermat first drew the axes and then graphed his points on those axes. Fermat's work was not published until his death in 1665. Fermat illustrates how to sketch a point on a locus in the first quadrant. Although it still had no negative abscissa or ordinate, this orthogonal system looks more like our present-day Cartesian system (Merzbach & Boyer, 2011). Based on the work of these two French mathematicians during the 17th century, the

English mathematician Isaac Newton (1642–1727) also worked and improved on this branch of mathematics. Newton modified the coordinate system to look more like what we have today (Merzbach & Boyer, 2011). He authored one of the early published works depicting an orthogonal coordinate system. In this work, he also introduced the negative axes, thus expanding the graph to four quadrants (Descartes and Fermat had no negative axes).

It is also well known from literature that different mathematical concepts/ideas are used in different branches of mathematics. For instance, Stillwell (2010, p. 109) points out that "around 1630, both Fermat and Descartes realised that geometric problems could be translated into algebra by means of *coordinates*, and that many problems could then be routinely solved by algebraic manipulation". Therefore, the use of either symbols or variables would assist in marrying these branches. For instance, to find the length of a line segment, gradient and equation of a line, the mid-point of the line, all require the use of variables and constants. René Descartes in the 17th century, modified Francois Viète's ways of writing, i.e. first letters of English alphabet for known quantities (constants) *a, b, c,…* and last letters for unknown quantities (variables) *z, y, x…* (Merzbach & Boyer, 2011; Stillwell, 2010). Around 17th century, the third development stage of Algebra was observed to have been used most in mathematics. The three developmental stages of Algebra include: Rhetorical stage- everything was written in words and sentences; the Syncopated stage- some appropriate abbreviations for use in mathematics texts were introduced and the Symbolic stage- use of full symbolism (Katz, 2007). In addition, the Theorem of Pythagoras is used to find the length of the line given two points.

The concept of gradient/slope/tangent is observed to have been used by Egyptians 4000 years ago. They used the idea when they wanted to construct the pyramids because they wanted to maintain the uniform slope for the faces (Merzbach & Boyer, 2011). In order to measure the steepness of a straight line, the ratio of "rise" to the "run" is used as Fermat, Descartes, Leibniz and Newton separately in the 17th century introduced this way (Stillwell, 2010) however, in Egypt, it was customary to use the reciprocal of this ratio (Merzbach & Boyer, 2011). The concept of linear equations was also used by Egyptians around 3000 B.C. (Burton, 2011). The theory of parallel lines in the Age of Leibniz and Euclid is outlined in different sources (e.g. Burton, 2011).

2.3 Benefits of using history of mathematical concepts in mathematics classroom.

Two notable factors that affect the learning of mathematics are motivation and interest towards the subject. The use of history of mathematical concepts in the classroom is therefore considered as a motivating factor to learners in their learning and study of mathematics (Burns, 2010; Farmaki & Paschos, 2007; Fauvel & van Maanen, 2002; Goktepe & Ozdemir, 2013; Jankvist, 2009). It is observed from these scholars that learners learn better if they are first motivated, and the use of history of mathematical concepts in the learning process may help learners become motivated and want to learn and explore more. Going through the history of mathematics, one can easily see the relationship between topics – like algebra and geometry – and how some concepts were invented. Connecting concepts, making sense of the ideas and justification of some terms or symbols used in mathematics are important pre-requisites for proper understanding of the mathematical concepts (Burns, 2010). The geometrical presentation of some concepts could motivate learners as they can see the concepts physically presented.

In addition, it has been found out that those learners who had undergone the lessons that incorporated the history of mathematical concepts performed significantly better than those who did not (Horton & Panasuk, 2011). This means that using history of some mathematical concepts in the classroom could help learners understand the concepts better, as some terms and ideas used in mathematics are classified and justified. The history of mathematical concepts may help learners appreciate how mathematical ideas have developed in the past and evolved over time. Goktepe and Ozdemir (2013) assert that the use of history of mathematical concepts could help learners see the stages of mathematics until the present day, and this could aid learners' understanding of concepts.

Furthermore, exposing learners to the history of mathematical concepts could help them minimise problems encountered in the learning process. They could improve their reasoning abilities, as they tend to note the perseverance earliest mathematicians demonstrated to invent the concepts (Barnett et al., 2013). So, learning mathematics through its history encourages learners to improve their critical thinking. It is also believed that using history to learn mathematics would encourage learners not to learn through memorisation (grasping the concepts as they are without critically examining them) and regurgitation (giving out basing on how somebody memorised it during a test or examination) – which results in poor understanding of the concepts but instead, learners should be active participants (Mac an Bhaird, 2009; Yee & Chapman, 2010). The active role learners take is what Barnett et al. (2013, p. 4) considered as the "read, reflect, respond" approach to the historical information of the concept.

It has also been observed that if historical information is used in classroom, learners might be able to compare and contrast the current knowledge to the past knowledge. Fauvel and van Maanen (2002) suggest that historical sources may help learners to put into perspective the current representational systems as just one of many possible ways of performing operations and handling and communicating mathematical concepts/ideas. It is also believed that learners will be likely to "appreciate the crucial role representations play in the inception and evolution of ideas" (Fauvel & van Maanen, 2002, p. 294). The knowledge learners might acquire through these activities would help them to be creative in the development of new ideas in mathematics. In addition, historical sources help learners become more aware of the creative processes that the mathematicians demonstrated in doing mathematics and be able to enrich their mathematics literacy (Bütüner, 2015; Jankvist, 2009).

2.4 History of mathematics versus attitude towards learning mathematics

Jankvist (2009) argues that history of mathematics may "give mathematics a more human face and perhaps make it less frightening" (pp. 237–238). Liu (2003) as cited in Goktepe and Ozdemir (2013, p. 126) found that history of mathematical concepts is important as it allows learners to "see the obstacles experienced in the development of mathematics in the past and to see difficulties encountered in present." On the other hand, Farmaki and Paschos (2007, p. 86) argue that, "[t]he historical analysis aims to present both the obstacles encountered in the development of various concepts and the ideas and methods by which these obstacles have been overcome historically." From the above arguments, one might agree with Jankvist (2009, p. 239) that "mathematics is a discipline that has

undergone an evolution and not something that has arisen out of thin air." Therefore, historical sources and information have the ability to enlighten learners that mathematical concepts were invented and modified, and that the knowledge could enable learners to adopt a positive attitude towards mathematics and contribute to their learning (Bütüner, 2015; Liu, 2003; Mac an Bhaird, 2011; Marshall & Rich, 2000; McBride & Rollins, 1977). The problems they encounter these days in their learning of mathematics could enable them to change their approach to learning and possibly find alternative ways of approaching lessons and mathematical problems.

The use of historical sources and information has a potential to also enlighten learners "not to get discouraged by failure, mistakes, uncertainties or misunderstandings," (Tzanakis & Arcavi, 2002, p. 207). This knowledge would enable learners to select more appropriate strategies to overcome their learning difficulties that may arise (Farmaki & Paschos, 2007). Learners' understanding of mathematical concepts is enhanced if mathematical ideas are presented in different forms, and the study of history of mathematical concepts may play a role in this respect. It has been noted that the use of historical sources in the classroom "may provide a more authentic or more complete picture of what mathematics is" (Jankvist, 2009, p. 244). On the other hand, Jankvist, Mosvold, Fauskanger and Jakobsen, (2015) suggest that using original historical sources could assist in understanding the subject matter itself better than just considering it as a finished product. The historical approaches could provide opportunities to experience different presentations of the same mathematical idea and thereby enhancing deeper understanding among learners.

2.5 Challenges of integrating history of mathematical concepts in mathematics classroom

Although many argue that history of mathematics may enhance teaching and learning, there are some limitations and challenges concerning the use of history of mathematical concepts in the mathematics classroom. These challenges could arise during the time of preparation, finding relevant source materials to be used in the classroom and implementing it into the classroom.

It has been observed that teachers would find it hard to find original and relevant materials because of language used (Bagni, 2000). Many of the historical materials used these days have been translated many times to make sure that the current generation is able to understand the mathematical ideas communicated. So, some original ideas might not be carried over to new generations.

Implementation of history of mathematical concepts might require teachers or implementers to be knowledgeable in the discipline (Tzanakis & Arcavi, 2002). It is believed that appropriate education is required to do this instead. However, many teacher training programs do not incorporate history of mathematics, and there is no provision that the in-service teachers would be trained in the history of mathematics in Malawi.

Tzanakis and Arcavi (2002) present several objections to the incorporation of history of mathematical concepts in curriculum and classroom. These are points that might affect the implementation and learners' understanding of concepts. Tzanakis and Arcavi (2002)

categorise these objections into philosophical and practical. The philosophical objections include:

> "History is not mathematics; History may be tortuous and confusing rather than enlightening; learners may have an erratic sense of the past which makes historical contextualisation of mathematics impossible without their having had a broader education in general history; Many learners dislike history and by implication will dislike history of mathematics, or find it no less boring than mathematics; Progress in mathematics is to make the tackling of difficult problems a routine, so why bother by looking back? and History may be liable to breed cultural chauvinism and parochial nationalism" (Tzanakis & Arcavi, 2002, p. 203).

They also suggest practical objections as: "Lack of time; Lack of resources; Lack of expertise; and Lack of assessment" (Tzanakis & Arcavi, 2002, p. 203).

2.6 Review of related research

Some studies have been conducted in various countries such as United States of America, Norway, Singapore, Netherlands, Denmark, Portugal (Fauvel & van Maanen, 2002) that investigated the role and integration of history of mathematical concepts in classrooms. Most of the studies the researcher came across were conducted at tertiary level and very few at secondary school level (e.g. Clark, 2012; Burns 2010; Haverhals & Roscoe 2010). For instance, Clark's (2012) study aimed at understanding school mathematics concepts for prospective mathematics teachers. She considered prospective mathematics teachers because they would be called to teach the topic in future, so they should be well equipped with knowledge of the concepts. She used the concept of completing the square of quadratic equations using Al-Khwarizmi's approach. She found out that studying history of a mathematical concept could be beneficial because it helps to deepen understanding of the concept in teachers and on how they can improve their pedagogical knowledge.

Burns (2010) conducted a study to investigate the views of US pre-service teachers concerning the benefits of using history of mathematics in learning and found that these pre–service teachers felt that it is important to expose the high school learners to the history of mathematics as they might be motivated in mathematics. In addition, the participants suggested that the knowledge of history of a mathematical concept would expand to more learning styles as some learners might learn through the use of stories and enable them to make connections between concepts.

Haverhals and Roscoe (2010), on the other hand, investigated the use of the history of mathematics as a pedagogical tool for the teaching and learning of mathematics. The study sought to investigate the merits of employing an historical approach through the teaching and learning of the integral of secant concept to a group of 16 undergraduate students at the University of Montana. They found out that the challenges of using history in mathematics teaching could be minimised through the use of historical approach as it provides the "crucial sources of inspiration, insight and motivation" (p. 353).

Goktepe and Ozdemir (2013) conducted the study on the grade 7 learners to find out the use of history of mathematics in teaching and learning processes of mathematics. Grade 7 learners were taught the concept of square root from a historical perspective and later a comparison was made with the contemporary methods of finding the square root of a number. They found out that using activities from history of the mathematical concept attracted the learners' interest and that learners have an opportunity to compare different methods of solving problems.

2.7 Theoretical connections

Using history of a mathematical concept in the teaching and learning of mathematics is not new. There is a broad literature available that concern the different purposes of using history in the mathematics classroom, curriculum or textbook. This section discusses how history can be integrated into the classroom and how learners might construct self-knowledge from the historical knowledge acquired.

2.7.1 The categories of integrating history of mathematical concepts in mathematics classroom

Since the use of history of a mathematical concept in mathematics classroom is not new, many researchers have reported on how to incorporate the history in the classrooms either at tertiary, secondary school or primary school levels, and they present different arguments for using history in the mathematics education. These arguments have been categorised into two: arguments referring to history as a tool and arguments referring to history as a goal (e.g. Jankvist, 2009). Jankvist (2011, 2009) theoretically suggests that using history-as-a-tool arguments concerns with learners learning of the inner-issues in mathematics (mathematical ideas, concepts, theorems, arguments, proof techniques and methods). Inner-issues relate to mathematical content or subject matter of a specific teaching module given to the learners.

According to Jankvist (2011, 2009) four useful ways that concern the use of history as a tool include: (a) an aid in the teaching and learning of mathematics inner–issues. In this case, history is considered as a vehicle for teaching and learning of inner-issues in

mathematics, (b) a motivational or affective tool. In this respect, history helps to sustain learners' interest and excitement in the subject and historical knowledge of the concepts may give mathematics a more human face and perhaps make it less frightening (c) a cognitive tool. The cognitive tool deals with the provision of different points of view of mathematics or the mode of presentation of different mathematical ideas or concepts. Jankvist (2009) argues that history cannot only help to identify problems learners encounter in understanding the concepts or ideas but also to help overcome them by providing essential clues which may specify the nature of the knowledge to be taught and learnt and explore different ways of access to that knowledge. and (d) an evolutionary argument (the recapitulation argument or historical parallelism). The evolutionary argument concerns with the claim that there can be no learning and teaching of mathematics without its history.

On the other hand, using history of mathematical concepts as a goal is concerned with showing learners:

> that mathematics exists and evolves in time and space, that it is a discipline that has undergone an evolution over millennia, that human beings have taken part in the evolution, that many different cultures throughout history have contributed to the development of mathematics, and that these cultures have had an influence on the shaping of mathematics as well as vice versa (Jankvist, 2011, p. 348).

2.7.2 Incorporating history of mathematical concepts in mathematics classroom

There are three essentially distinct categories of incorporating history of a mathematical concept in the classroom. For example, one could think of illumination approaches, the module approaches and the history-based approaches (Jankvist, 2009, 2010, 2011).

In an illumination approach, teaching and learning of existing mathematical concepts is supplemented or spiced up by the historical information of the concepts. The information could be on the names of mathematicians, the dates on which the concepts were developed, famous works and events, time charts, biographies, famous problems and questions (Jankvist, 2009).

In the module approach, instructional units are devoted to history and, quite often, they are based on cases. These may vary in size and scope, from small modules closely tied to an existing curriculum and longer modules less tied to curriculum content to full courses on the history of mathematics within a mathematics program (Jankvist, 2011). The small modules focus much on the content to be included in the curriculum and that could be covered in the classroom. For instance, Tzanakis and Arcavi (2000) as cited in Jankvist (2009, p. 246) consider this as "historical packages," which are collections of "materials narrowly focused on a small topic, with strong ties to the curriculum, suitable for two or three class periods, ready for use by teachers in their classroom". Jankvist points out that the historical packages could be implemented in different ways such as through textbook studies, through readings of original sources, or through student projects, worksheets and historical problems.

On the other hand, Jankvist (2011, p. 348) points out that "the history-based approaches are those directly inspired by or based on the development and history of mathematics". The approach could be used as a guide to order and the way in which the mathematical topics are presented. Jankvist (2011) suggests that a drawback of this approach is that the historical development is not forced to be discussed openly.

The assumptions that Jankvist built his work upon were both theoretical and empirical. Several researchers have also carried out empirical studies on the use of history of mathematics. For instance, Jankvist (2010) carried out an empirical study of using history as a 'goal'. On the other hand, Clark (2012) carried out a research on the history of mathematics mostly basing on the illumination approach. Other researchers who have conducted the studies on the use of history of mathematics in the classroom did not openly outline how they used history (either using history as tool or as a goal) and the category of approaches of incorporating history they considered.

2.7.3 Constructivism

The study is also based on constructivist theory of learning in mathematics education. It is assumed that this theory enables learners to "learn mathematics in a more active way and construct, step by step, their own mathematical knowledge" (Fauvel & van Maanen, 2002, p. 75). Constructivism in general is a theoretical perspective on knowledge and learning that has gained international recognition as a theory which has much to offer to mathematics education (Thompson, 2000). Constructivism considers that knowledge is not passively received but actively built up by the cognising subject (von Glasersfeld,

1995; Yackel et al., 2011) and the cognising subject is considered as the learners themselves. Stuart et al. (2009) on the other hand, pointed out that the main idea of constructivism is that each learner constructs his or her own knowledge in a unique way and "any kind of learning implies a construction of new knowledge in the individual" (Mosvold, 2006, p. 23). Mosvold further suggests that the process of knowledge construction should be rooted in an individual learner for the notion of learning by the learner to provide meaning. If this construction happens in an individual learner, it is referred to as radical constructivist view.

Active construction of knowledge by learners according to the requirement of radical constructivists need learner-centred and discovery-oriented learning processes (Liu & Matthews, 2005). On the other hand, von Glasersfeld (1995, p. 1) defines radical constructivism as "an unconventional approach to the problems of knowledge and knowing". It is unconventional because different approaches might be used or followed as long as an individual learner is able to construct his or her own meaning of the concept. He further points out that radical constructivism,

> starts from the assumption that knowledge, no matter how it be defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience (p. 1).

In Malawi and other countries, mathematics teachers are encouraged to use a variety of approaches such as learner-centred or discovery-oriented. However, learners might have different perceptions on the use of these learner-centred or discovery-oriented approaches. It has been observed that using history of mathematical concepts in the classroom might motivate learners (e.g. Fauvel & van Maanen, 2002) and be willing to participate in the learner-centred or discovery-oriented approaches.

To achieve uniformity of knowledge construction of certain mathematical concepts and to achieve intended goals of education as per curriculum, teachers need to guide the learning process. Teachers need to be aware that knowledge is not fixed but fluid and expanding (Stuart et al., 2009), therefore, learners might be able to construct different meanings from the concepts. This has been noted in the historical development of some mathematical concepts such as the development of coordinate geometry where different mathematicians played different roles to refine the concepts in the topic. The area that could be shown to learners that knowledge is not static is the improvement on axes to have both positive and negative values as used today.

Stuart et al. (2009) also suggest that teachers who base their methods on constructivist learning theories try to give individual learners a considerable amount of freedom to learn in their own way and at their own pace so that each individual learner acquires specific skills and knowledge from each mathematical concept. Learners could construct different knowledge from the same mathematical concepts but the other knowledge that enhance the development of new concepts is also allowed. For instance, Von Glasersfeld (1989, p.

10) concludes that "radical constructivism does not suggest that we can construct anything we like, but it does claim that within the constraints that limit our construction there is room for an infinity of alternatives".

Though radical constructivist view puts more emphasis on individual learning, some aspects of social learning are also noted. This means that though learners might work in pairs or groups, the focus is on how individual learners understand mathematical ideas, conceptualise them and make meaning out of that (Anderson, Reder & Simon, 1998). So, this approach could support learners' co-construction of knowledge considering their struggles in the knowledge construction. It has been observed that in the past, knowledge construction was not easy as mathematicians also struggled and took many years to refine their work. Fauvel and van Maanen (2002, p. 148) suggest that "the importance of teachers having some knowledge of the history of mathematics … may help them to help their learners overcome some important difficulties which arise in the mathematics classroom". Similarly, the knowledgeable learners in the history of mathematical concepts might be able to have a realistic chance of actually working with the concepts and be able to construct meaning from them.

Furthermore, Thompson (2000) suggests that the constructivist approach to teaching and learning of mathematics does not suggest clearly the instructional strategies that teachers can adopt. Constructivists suggest that learning and understanding of concepts may suggest a new set of goals for the classroom. Therefore, constructivist approach to teaching and learning in mathematics enables teachers to understand that teaching

mathematics provides learners with the opportunity and encouragement to construct powerful mathematical ideas for themselves and to come to know their own ability as mathematics thinkers and learners (Moyo, 2014; Stuart et al., 2009) which may also be motivated by the use of history.

In addition, constructivist theories consider that mathematics learning is not "approached from a point of view that says the learner knows nothing and the teacher must fill him or her up with truth" (Moyo, 2014, p. 20). Thus, learners should be able to use their thinking processes to create their own understanding of concepts and be active and reflective in their acquisition of mathematical knowledge (Fried, 2007). This implies that although constructivist approach does not suggest an instruction strategy, it could suggest the role of the teacher in classroom discussions.

Teachers as facilitators should understand that "construction of knowledge is seldom a construction of genuinely new knowledge. It is normally more of a reconstruction of knowledge that is already known to the general public, but new to the individual" (Mosvold, 2006, p. 24). According to the radical constructivist, a facilitator is an individual who provides the learning environment conducive for the learner to learn on his/her own. The teacher, therefore, should initiate constructive activities because minds work by themselves sometimes. It is also important for the facilitator to take into consideration the background of learners as different learners would perceive history of mathematical concepts differently, therefore, knowledge construction basing on this would be different as well.

The related studies reviewed indicate that they were conducted after the history of mathematics was introduced into the curriculum and most of them are the follow-up studies to find out the effects of either using history in classroom or in textbooks to both the teacher and the learner, and few have targeted secondary school learners. Some of these studies used different theoretical frameworks as there are many theories of learning in mathematics. Most of the studies on constructivism consider the social constructivism and how learners collaborate to construct mathematics knowledge. This study intended to find out how knowledge is constructed in individual learners (radical constructivist's view of learning) through the knowledge acquired from the use of history of the mathematics concepts they are learning. In addition, in Malawi the use of history of mathematics in mathematics education is not included in the secondary school curriculum and no research has been conducted to investigate the use of history of mathematical concepts in teaching and learning mathematics. Hence this study was conducted to investigate the use of history of mathematical concepts in mathematics learning in secondary school where history of mathematics has not been included in the mathematics curriculum.

2.8 Summary of the chapter

The review of the literature has shown that many studies have been conducted on the role of history of mathematics in mathematics teaching and learning. There are different reasons history of mathematics is used in the mathematics education, for instance, to enhance learners' motivation and to help them deepen their understanding of mathematical concepts. Different mathematical concepts are used basing on the level of

education and on the reasons of using history in mathematics classroom. The mathematical topic of the study has been briefly discussed in terms of its development. The chapter has also discussed the reasons of integrating the history of mathematical concepts in mathematics classroom, how and when it can be incorporated and the challenges that might affect its inclusion in the mathematics classroom. The chapter has also outlined the categories and approaches of using history of mathematical concepts in mathematics teaching and learning. The study has considered the use of illumination approaches. The study also considered the links between the categories of integrating history and how learners construct mathematical knowledge basing on the radical constructivist learning theories in mathematics. The chapter to follow discusses the research design and methodology used in the study.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

The preceding chapter discussed the related literature to the study and the theoretical framework for the study. This chapter discusses the research design and introduces the participants and sampling, research instruments, data analysis and interpretation, and the ethical considerations of the study.

3.2 Research design: A case study

The current research is qualitative in nature, and a case study approach to research was considered. A case study is defined as "an empirical inquiry that investigates a contemporary phenomenon within its real life context; when boundaries between phenomenon and context are not clearly evident; and in which multiple sources of evidence is used" (Yin, 1989) as cited in (Glatthorn & Joyner, 2005, p. 43). On the other hand, the case study approach aims at studying intensively the background, current status, and/or environmental interactions of a given social unit such as an individual, a group, an institution or community (Creswell, 2012). Cohen, Mannion and Morrison (2007) also suggest that some case studies are of a single situation, for instance, a single social group, a single class, or a single school. In this study, the case study focused on a teacher as an individual and a group of learners in the classroom. In addition, case study allowed the

researcher to carry out a deep investigation and analysis of a single entity (Clack & Carter, 2004; Creswell, 2012). This means that a case study is also concerned with exploring, describing and explaining a phenomenon.

On the other hand, a case study can use a small group in order to learn more about social realities in a particular context. It allows the researcher to probe with the necessary depth and recognition of the context and hopes to construct knowledge that will be applied to address the social problem (Cohen et al., 2007). Case study research enables the researcher to arrive at an understanding of a complex situation, in this case, the issue of using the history of a mathematical concept in classroom, and this can also add value to what is already known through previous research.

In addition, "the case(s) is/are bounded by time and activity, and researchers collect detailed information using a variety of data collection procedures over a sustained period of time" (Creswell, 2009, p. 227). The current study was restricted to time allocated to a topic chosen to be taught to Form Three learners and the collection procedures were observation, questionnaires and face to face interviews. Glatthorn and Joyner (2005) suggest that a case study research is a disciplined inquiry that is concerned with illuminating the meaning by using inductive processes (data interpretation). In addition, Cohen et al. (2007, p. 253) point out that indeed a "case study can enable readers to understand how ideas and abstract principles can fit together". The meanings are obtained through qualitative data analysis rather than using statistical generalisation.

The case study for the current research was motivated by previous researches within the area of study where this approach was used in various institutions and levels. For instance, Clark (2012) used a case study in her research to investigate the effects of history of mathematics on prospective mathematics teachers on the understanding of school mathematics concepts. She considered this research approach because she wanted to understand how this group of prospective teachers doing mathematics education would profit from solving quadratic equations from a historical perspective. A case study research was also considered by Goktepe and Ozdemir (2013). They employed this approach because they wanted to get opinions of grade 7 learners on the use of history of mathematics in teaching and learning processes of mathematics.

Similarly, Haverhals and Roscoe (2010) used case study research to investigate the use of the history of mathematics as a pedagogical tool for the teaching and learning of mathematics. Their study sought to investigate the merits of employing an historical approach through the teaching and learning of the secant concept to a group of 16 students at the University of Montana. This was also done in the classroom setting using the time allocated to the course. This approach was employed with the aim of intensively studying the behaviour of students during the time history of mathematics was used in teaching mathematical concepts.

Therefore, the current research also employed case study design to explore deeply the learners and teacher attitudes and beliefs towards the use of history of mathematical concepts in the mathematics teaching and learning. In this case study research, the

researcher wanted to have an in-depth study of the phenomenon and to generate detailed information about the issue, that is, the integration of history of mathematical concepts in the mathematics teaching and learning. On the other hand, the findings obtained from a case study in qualitative research might also enable the researcher to generalise to theory, but not necessarily to a larger population (Flyvbjerg, 2006).

3.3 Mathematics topic taught

The sample was taught the concepts from Coordinate Geometry also known as Cartesian Geometry. Coordinate Geometry as is presented in Malawian mathematics curriculum involve different learning targets. These learning targets include: calculating the length of a line segment, finding the midpoint of a line segment, finding the gradient of a straight line given two points, finding the equation of a line from the graph, stating the relationship between gradients and parallel lines, find the equation of a line through a given point and parallel to given line. The historical information of some concepts given above has been outlined in section 2.2. This topic was chosen from the syllabus because there is adequate historical background to the topic and on some concepts that would attract learners' attention if used in the classroom.

Therefore, teaching and learning of the chosen topic was spiced up by its historical information so as to enable learners know how these concepts were developed and dealt with. The historical information of the topic and some concepts were incorporated into the classroom basing on the illumination approach. In an illumination approach, teaching and learning of existing mathematical concepts in the curriculum is supplemented or

spiced up by the historical information of the concepts (Jankvist, 2009, 2011). The information was about the names of mathematicians, the dates on which the topic was developed, the part each of the mathematicians played in the development of the topic, time charts and biographies and why they developed the concepts. The teacher and the researcher discussed what historical information was to be used in the classroom before the learners were taught the concepts from the chosen mathematics topic. This was done so to give the teacher guidance as to how history of mathematical concepts might be used in classroom as he had not used it before. So, the teacher used the historical information discussed with the researcher as outlined in section 2.2 when introducing the mathematics topic to the learners and teaching of other concepts within the chosen mathematics topic. In the classroom, the teacher outlined the development of the topic. That is, when the topic was discovered, who took part and how some concepts were modified to look like today and the implications of the concepts in everyday activities.

Furthermore, in this qualitative case study approach, open-ended questions were used with the aim of collecting data on the learners' and teacher's perceptions about using history of the mathematical concept in mathematics classroom. The other aim of employing a qualitative approach was that the Malawian secondary school education system does not explicitly incorporate history of mathematics; hence some educators do have limited knowledge about the aims of using history of mathematical concept in classroom especially at secondary school level.

3.4 Participants and sampling

The participants that were targeted were the form three learners and their mathematics teacher from a Public Day Secondary School in Zomba Urban. The rationale for selecting these participants was: (1) some concepts in the chosen topic were already covered by them at lower level; (2) they have learned a lot of concepts in mathematics; (3) they may have improved their reasoning abilities; and (4) they might be able to question the existence of some concepts and look for proper justifications. Form three learners were required to respond to the questions in a questionnaire and interview. Teacher being the central in teaching was allowed to (apart from the normal way of teaching) conduct the lessons by incorporating historical information on the concepts. The researcher was there to observe the lessons and video recording the lessons in order to refer to the data again later. Both male and female learners were involved in the study. In addition, this group of participants was chosen because they were not preparing for the national examinations as was the case with form four learners.

Sampling refers to the process and techniques used to select the study participants. Sampling enables the researcher to generate data by working with a manageable and accessible group that is representative of a larger population (Cohen et al., 2007). Sample refers to a "subgroup of the target population that the researcher plans to study for generalising about the target population" (Creswell, 2012, p. 142). A convenient sample was used in this study and data were collected from a group of learners and their teacher who were based in one school in Zomba Urban. In conducting a research, it might be difficult to recruit teachers, learners or schools to participate in a study of this nature,

hence a convenient sampling technique was chosen for this study. In convenience sampling (which is non-random sampling method), "the researcher selects participants because they are willing and available to be studied" (Creswell, 2012, p. 145). Creswell (2012) does, however, suggest that a convenience sample is not necessarily representative of a population, but the sample selected might still provide useful information for answering the research questions and achieve the purpose of the study.

Fifty learners were given a questionnaire to respond to during the class time. These learners were given 30 minutes to respond to the questions on the questionnaire and after 30 minutes every learner handed in the questionnaire to the researcher. This was done so because the researcher thought that taking questionnaire home would allow the learners seek views from colleagues and other individuals and the responses to the questions would not be a true reflection of their attitudes and beliefs. Out of fifty learners who responded to the questions on the questionnaire, ten learners were selected for focus group discussion to represent the opinions of fellow learners. These learners were only selected basing on their willingness to take part in the discussion not basing on their open-ended responses from questionnaire.

3.5 Data generation

In a qualitative research, as some scholars argue, the term *data generation method* is preferred as opposed to data collection. According to Ndengu (2012), researchers argue that using the term 'data generation' means that participants' active and cooperative role is recognised in the research process as opposed to a passive role of information donors in data collection. The researcher visited the school at the time Coordinate Geometry was about to be taught to form three learners. Before the teacher started teaching the concepts from the chosen topic to the learners, the researcher and the teacher discussed some points on history of the concepts on the topic to be incorporated into the lessons. The topic was scheduled to be taught within a 2–week period.

3.5.1 Research Instruments

The study generated data through three different instruments: classroom observations, questionnaire and interview. There was a questionnaire for learners and another for the teacher. Ten learners, after responding to the questions in a questionnaire, were invited for interview. Different instruments were considered because a case study is not strictly a distinct method but employs other methods to investigate the phenomenon. Thus, a case study may, for instance, involve interviews and in-class observation (Clark & Carter, 2004). The researcher's role in the classroom was to observe the behaviour of learners and their teacher as history of mathematical concepts were being incorporated. The researcher had some knowledge on how the history of mathematical concepts could be incorporated into the mathematics classroom, but he tried not to interfere with the teaching and learning process while observing the lessons in the classroom.

3.5.2 Classroom Observation

Observation is considered as a suitable instrument that might be used to gain a rich picture of any social phenomenon – such as the behaviour of learners in a classroom – and the researcher gather firsthand information by observing people and places at a research site (Creswell, 2012). Classroom observations were considered because the researcher also wanted to observe the behaviour of both the teacher, the learner, and the interactions between them as history of mathematical concepts was used in the mathematics teaching and learning. In addition, observations were made for the reasons such as: (1) to find out the acceptance of learners to the historical information on the mathematical concepts; (2) difficulties learners and the teacher might encounter in the process of teaching and learning mathematics using its historical information; (3) to check whether the teacher might modify the instructional strategies basing on the historical information; and (4) how often can history be used in the classroom within the chosen mathematics topic.

Furthermore, classroom is considered as the learners' and teacher's natural social environment where the researcher can generate "live data" (Cohen et al., 2007, p. 396). In this case, observation also enabled the researcher to investigate the reality of the situation in the classroom, because what people do may differ from what they say they do.

3.5.3 Questionnaires

Two different questionnaires were used: teacher questionnaire and learner questionnaire. Open-ended questions were used in the questionnaires and allowed learners and their teacher to provide a more complete and comprehensive response basing on their understanding of the phenomenon.

3.5.3.1 Teacher questionnaire

A teacher questionnaire was used with the aim of generating data on teacher's attitudes and beliefs on the possible benefits of using history on learners' mathematical concepts understanding and attitudes toward mathematics, how and when to incorporate history in classroom and challenges of incorporating history of mathematical concept in teaching and learning mathematics in classroom. Questions in this questionnaire were partially similar to questions in learner questionnaire and was administered after the teacher completed teaching the learners the chosen mathematical topic. This was done bearing in mind the concept of triangulation. Creswell (2012) defines triangulation as "the process of corroborating evidence from different individuals (…), types of data (e.g., observational fieldnotes and interviews), or methods of data collection (e.g., documents and interviews) in descriptions and themes in qualitative research" (p. 259). Thus, two or more methods of data generation in the study of some aspect of human behaviour could be used (Cohen et al., 2007). Cohen et al. also advocate that methodology triangulation may involve the use of different approaches on the same phenomenon to be studied. This is done with the aim of generating rich data and enhancing the accuracy of study.

3.5.3.2 Learner questionnaire

A learner questionnaire was used to collect data on their attitudes and beliefs on the benefits of using history on their understanding of mathematical concepts, when and how to incorporate history and challenges of using history of mathematical concept on mathematics learning. Basing on the data collected from classroom observation on the incorporation of history of mathematical concepts in the classroom, a learner questionnaire was designed to generate other data from learners. A questionnaire was suitable in this case because many learners were involved in the study and it was not possible to interview all individual learners in order to get their attitudes and beliefs.

In this study, the learner questionnaire was administered after the completion of the topic they were taught using some historical information of its development. Learners were asked to respond to the questions in a questionnaire either in Chichewa or English. Different languages were considered because learners could have good points but would fail to express themselves if restricted to one language (e.g. English).

3.5.4 Interviews

Interview is a type of discussion that is carried out with the aim of getting the detailed information from participants on the phenomenon (Ndengu, 2012), and it was used to obtain additional qualitative data on the role of history of mathematical concept in mathematics teaching and learning. The interview method was considered so as to compliment to the questions in a questionnaire as some issues in the study were not addressed fully. Both teacher and learners were interviewed.

3.5.4.1 Teacher interview

The teacher was interviewed during the lesson planning because the teacher and the researcher discussed the information on the historical development of the topic and some concepts to include in the lessons. The questions that guided the open discussion have been provided in appendix F on page 106.

3.5.4.2 Learner interview (Focus group interview)

Learners were interviewed to obtain further information and clarification on learners' responses to the questionnaire questions. This was done to a group of 10 learners. The focus group was employed because according to Onwuegbuzie, Leech and Collins (2010), there is;

> "high face validity"; "create an atmosphere where more responses can take place"; "create a "safer" environment than can individual interviews" and "provide a place for participants to interact in such a way that personal issues and problems can potentially be solved" (p. 711).

These learners were asked to willingly participate in the interview. The assumption of selecting 10 learners was that their attitudes and beliefs would represent the whole group of learners involved in the study. The interview mainly centered on the incorporation and the challenges of implementation of history of mathematical concept in mathematics teaching and learning. The interview guide has been provided in appendix E on page 105.

3.6 Data analysis and interpretation

The qualitative data generated were interpreted using qualitative data analysis. For instance, all interviews and in-class observation recordings were transcribed and the transcripts were reviewed against the recordings to ensure accuracy. The investigation comprised of open-ended questions in which participants were expected to express their attitudes and beliefs regarding the use of history of mathematical concept in mathematics learning. Therefore, the data generated demanded inductive qualitative data analysis (Creswell, 2012). This process of inductive analysis involves "coding, categorising, and interpreting data to provide explanations of a single phenomenon of interest" (McMillan & Schumucher, 2010) as cited in (Moyo, 2014, p. 32). Qualitative data were categorised into themes (themes generated from the research sub-questions) that determined participants' perceptions towards the use of history of mathematical concepts in mathematics learning.

3.7 Ethical considerations

Ethics are normally concerned with beliefs about what is right or wrong, suitable or unsuitable, moral or immoral. In research, the ethical issues are concerned with the protection of rights and welfare of participants (Creswell, 2012). Therefore, the principal rule of ethics in research is that participants should not be harmed in anyway, either physically or mentally.

In order to abide by the ethical issues in research, the researcher first sought a letter of introduction from the department of Curriculum and Teaching Studies in the Faculty of Education at Chancellor College. This letter was given to the headmaster of the school where the study was conducted seeking the consent to go ahead with the study at that site. Since the study was conducted in the classroom situation learners' participation in the learning process was not interrupted. Learners for the interview were asked to willingly take part and this was done soon after they finished normal classes for the day.

Learners were also informed about the purpose of the study and that they were free to participate or refuse to take part in the study or withdraw anytime they feel not comfortable with the study more especially on responding to the questionnaire.

Learners were also assured of confidentiality of the study that the data generated would solely be used for the academic purposes. Learners and the teacher were also assured that their names would be replaced by pseudonyms. In addition, no specific details of the school where the study was conducted has been given in the thesis.

3.8 Limitation of the study

It was observed that some learners did not respond to all questions in the questionnaire. Fortunately, the data I got from other learners were enough to answer the research questions because each question in the questionnaire was responded to.

3.9 Summary of the chapter

This chapter dealt with the research design and methodology. The study was designed as a qualitative case study in which qualitative data were generated through the use of questionnaires, classroom observation and interview to form three learners and their mathematics teacher. Data analysis was inductive in nature. To generate qualitative data, open-ended questions were used and participants' (learners and teacher) responses were categorised into themes. These themes were used to determine participant's beliefs and attitudes or perceptions towards the use of history of mathematical concepts in mathematics teaching and learning. Ethical considerations and limitation of the study were also taken into consideration. The next chapter looks at the results of the study and the discussion of the findings.

CHAPTER FOUR

RESULTS AND DISCUSSION OF THE STUDY

4.1 Introduction

The preceding chapter was about the research design and methodology used in this study. This chapter presents the results of the study and their discussions. Results and discussions of research findings are presented in thematic sections basing on the research sub-questions. Data generated from three instruments were analysed and the findings have been presented according to their commonality. The four research questions are used as headings on each subsection.

4.2 Benefits of using history of mathematical concepts in developing mathematical concepts in learners

The analyses of teacher's and learners' responses have been grouped into different categories and these categories were inductively selected from the data collected. This was achieved because categories were drawn by considering specific units of data so that the general issue is arrived at. These include; mathematics development (evolution) and concept understanding, learners' creativity, strategies used in mathematics learning and reminder/ reference in mathematics learning.

4.2.1 Mathematics development (evolution) and concept understanding

Results from learner questionnaire have indicated that using history of mathematical concepts in mathematics learning is useful. History of mathematical concepts provides useful background knowledge on the development of mathematical concept itself. For instance, out of 50 learners involved in the study, 35 learners indicated that history of mathematical concept would help to highlight some important information on the development of the concept. The information referred to might be about how the mathematical concepts came into existence. For instance, some learners suggested that using history of mathematical concept in classroom would highlight to them that mathematics has been in use for many centuries and enable learners to do a lot in mathematics learning so that they should understand these concepts. Here is what some learners had to say about the role of using history of the mathematical concept on the development of mathematics:

> *It simply reminds us that mathematics has been there since in the past and we should focus on it;*
>
> *It helps one to know how the topic started and the formulae;*
>
> *It helps to know why they did that and how they did that;*
>
> *Because we are able to see what have been changing in mathematics;*
>
> *It helps us to understand the past and the future events;*
>
> *By knowing the problem that has been faced by early people and finding solution for use to be successful.*

In addition, the findings have also indicated that learners would acquire general information about the people involved in the development of mathematical concepts learners are learning today. The responses also indicate that learners have acquired the knowledge about the country of origin of the mathematicians and who discovered what. Typical learner responses on the matter are presented as follows:

> *You can easily know where that kind of mathematics came from, who invented it and the country which inventors came from and which year;*
>
> *I had no idea of who started the mathematical problem;*
>
> *I had no idea of who invented the concepts of mathematics;*
>
> *I have known the people who were involved and ways of solving mathematical problems;*
>
> *We know the places at which some mathematics studies originate.*

Learners' responses also indicate that they find the use of some historical information of the mathematical concepts they are learning supportive to their learning process as was also observed in the teacher questionnaire. As such, learners would use the knowledge and skills of ancient mathematicians to sharpen and develop their own reasoning skills and become critical thinkers. These learners would emulate what ancient mathematicians have been doing and would also be able to construct meanings from the concepts.

Here is what some of learners had to say:

> *By using the history of mathematics helps to acquire some knowledge or skills that other people used;*
>
> *It is important because history of mathematics helps a person to know some of the information and ideas that people put in this subject mathematics;*

It is important because we know that some people had good skills and that others could not have in the future;

By knowing where mathematics started, where we are today concerning mathematics and know where we are going with mathematics. This will give me starting point in the future when I will desire to study mathematics and come up with my own theorem about different ideas of mathematics.

On the other hand, the teacher pointed out that history of the topic or concepts might enhance learners' learning process. The knowledge that it took a long time for mathematicians to develop and refine mathematical concepts and that different people were involved in order to modify the concepts so that the present generation would be able to understand and use these ancient mathematical ideas would help learners to improve their reasoning skills in mathematics. This also means that the learner should understand that nothing comes out easy but requires an effort. It might be observed from the teacher's response that learners should be focused on whatever they are doing as has been in the development of these concepts. The challenges faced in the past and how they were dealt with would help some learners to develop their confidence that they can also do mathematics provided they put an effort to it. For instance, the teacher said:

Learners appreciate that before them they were some people who struggled to make the understanding of mathematics concept become simpler hence motivate them to keep on working hard

4.2.2 History of mathematical concept and learners' creativity in mathematics

The findings from learner responses have indicated that some learners would use some of the historical information of the concepts in the learning process. Some learners suggested that they would become critical thinkers and be creative. Other learners also

suggested that they would try to minimise the mistakes made in the past and try to improve on the concepts basing on the current situations. For example, 30 learners suggested that knowledge of how some mathematical concepts were developed would enable them to correct the mistakes made in the past by analysing the entire processes in which the concepts were developed. It has also been observed that some learners' responses have also indicated that they might come up with new improvements to these concepts to solve current mathematical problems. These improvements could be enhanced if ideas from the ancient mathematicians are used after following how they managed to come up with some concepts.

Here are some of the common learner responses on this category:

> *It will help the present people not to repeat the same mistakes that were made in the past;*
>
> *By making us not to repeat the same mistakes that people made before;*
>
> *Helps us not to do the mistakes that the old people did in the past years;*
>
> *To avoid the mistakes they had made and also to predict the future;*
>
> *History of mathematics helps one to correct some mistakes;*
>
> *To correct faulty thinking because it helps us to differentiate how things were in the past and also how it is in nowadays;*
>
> *For easy not to repeat past mistakes but to desire styles not to copy, to make our own things not perpetuate things you know;*
>
> *A person becomes creative thinking.*
>
> *In order to improve them and never face the problems again*

The learners' responses also indicate a belief that using historical information of a mathematical concept in classroom would help them become active in the mathematics learning processes. In this case, learners would be able to analyse how the mathematical ideas have been coordinated and this would enable them to develop other mathematical concepts. It is then assumed that this use of historical information of mathematical concept would enable learners to be actively involved in mathematics classrooms in knowledge construction.

Not only did learners suggest that they would be creative, but also the teacher suggested the same. The teacher pointed out that if the history of some mathematical concepts is incorporated into the lessons, then some learners would borrow the ideas and use them for their learning and knowledge construction. For instance, the teacher said:

> *Some learners have borrowed a leaf on working hard till they get correct solutions (a slight change on their mindset towards positive)*

4.2.3 History of mathematical concept and the use of different learning strategies.

The analysis of learners' responses indicate that half of the learners who responded to a questionnaire pointed out that historical information would help them to look for different learning strategies in mathematics. This indicates that understanding of mathematical ideas requires the use of different learning strategies as different learners understand mathematical concepts differently.

It has also been found out that the historical knowledge of the concepts would enable learners to identify methods that are suitable for the problems they are facing. In addition, it has been observed that concept understanding is enhanced if individual learners are able to identify and use different learning strategies in mathematics. Mathematical knowledge is not constructed out of nothing but requires different skills obtained from different sources. Therefore, history of the mathematical concept would play that role of enhancing understanding of concepts as it illuminates learners to look for alternative ways of doing mathematics. Learners would also come across different activities from different mathematicians as they were developing the concepts and the variety of skills they used.

Here are some of the learners' responses from the questionnaire:

> *I can link what people did in the past with todays and apply them in solving mathematical problems. For example, if I fail solving mathematics with today's methods I can use what people did in the past and come up with the answer to the problems;*
>
> *These will help us for example, when we know what others did we will be able to link what others did and what we are doing to invest new ideas in mathematics which will be helpful;*
>
> *It will also help people to find alternative ways of discovering the concepts than those were using in the past;*
>
> *It helps one to have an idea on how the people used their strategies;*
>
> *It will help me to follow their footsteps by knowing what they did and how*
>
> *they did and they managed to solve those mathematics problems.*

4.2.4 History of mathematical concept acts as a reminder or reference in mathematics learning.

It has been found that both learners and the teacher considered the use of history of the mathematical concepts to act as a reminder or a term of reference in mathematics learning. Learners are subjected to a lot of concepts in mathematics and other subjects, they can easily forget them if they come back to them sometime later and/or during examinations. In addition, if learners are not clear on some areas in mathematics, they may refer back to the original sources or changes that have taken place in mathematics for further understanding of the concept. So, it is believed that learners' memory could be ignited if they recall either the names of mathematicians or stories behind them or any historical aspect on the concept. This is also true as some theorems and topics are named in honour of the people involved in their development.

Some of the responses from learners are given as follows:

> *Ngati pali mayina vuto likabwera timakumbukira mwachangu ndipo timakumbukiranso njira (if there are names of inventors/discoverers, then when a problem comes we can easily remember the concept in consideration that time and even the method of solving the problems);*
>
> *History of mathematics has opened our mind in terms of thinking quickly or active;*
>
> *History of mathematics helps in the understanding of mathematics for easy solving of problems given now.*

On the other hand, the teacher agreed with the learners' suggestions that using history of the mathematical concept would *act as term of reference when solving certain problems.*

4.2.5 Summary

The findings from the analysis of learners' and teacher's responses have been grouped into different categories. These include (a) mathematics development (evolution) and concept understanding, (b) learners' creativity, (c) strategies used in mathematics learning, and (d) reminder/reference in mathematics learning.

It has been noted that history of mathematical concept enables learners to have general information on how the mathematical ideas were developed and how that enables them to construct their own mathematical knowledge as they get encouraged by other mathematicians' work. The struggles ancient mathematicians met and how they overcame them, give learners confidence that they too can do mathematics no matter how long it might take to understand the concepts.

It has also been found out that learners would be able to construct their mathematical knowledge since their reasoning and creative skills have been enhanced with the use of history of mathematical concepts in classroom. Some learners would be able to emulate what ancient mathematicians were doing and help their mathematical concepts understanding. In addition, the analysis has also shown that history of mathematical concepts might enable learners to use a variety of learning strategies. Concept understanding is enhanced with the use of different learning strategies in classroom. Reviewing literature on how the mathematical concepts were developed enables learners to find additional strategies of solving mathematical problems.

Finally, the analysis has also shown that mathematics learning needs to have a basis, that is, the point of reference. So, the history of a mathematical concept plays this part. Learners would be able to refer to what was done in the past and that would help them to recall some concepts easily.

4.2.6 Discussion

The findings of the study have revealed some possible benefits of using history of mathematical concepts in mathematics learning. It has been observed that for learners to construct knowledge from the concept, they should first understand the nature of the subject – in this case, the historical development of the concept. They should be able to recall and link different concepts they have studied in the entire time they have been learning mathematics. The findings have also shown that the historical information of mathematical concept might enable learners to know that mathematical ideas are not static but are continually being developed and improved. These findings are consistent with those of Mac an Bhaird (2009), Charalambous, Panaoura and Philippou (2008) and Ozdemir, Goktepe and Kepceoglu (2012). The findings are different from how most people perceive mathematics as a subject full of accumulation of ideas; learners find similar mathematical concepts in most of the textbooks they have come across in their learning of mathematics. As such they might think that these mathematical concepts could not be improved/modified.

On the other hand, changes that have taken place during mathematical concept development enable learners to have the general knowledge on the challenges ancient mathematicians encountered, the time they took to develop concepts and how they overcame the problems. As was observed by Jankvist (2009), one of the aims of using history of mathematical concepts in mathematics education is to use it as a motivating factor for learners in their mathematics learning and the study of mathematics. Therefore, this knowledge of history of mathematical concept development would enhance learners' mathematics learning, since, in the process of going through historical development, they would come across different struggles ancient mathematicians encountered and how they built knowledge from that.

The knowledge of the stages in which a concept was developed could also help learners build confidence in doing mathematics, because what they encounter today shares similarity with what has been experienced by other people in the past. This is observed as the mathematical knowledge constructed is not first considered as right by either their mathematics teacher or colleagues but gets accepted after some time. This indicates that knowledge construction could be gradual and sometimes it could be fast as was in the past. For instance, Katz (2007) points out that Cayley gave an axiomatic definition of a group, but his definition did not make it into the mathematical mainstream for another 30 years. It was only accepted after it was rediscovered later by other mathematicians. Therefore, learners' struggles in doing mathematics are not strange; similar struggles have been there many centuries ago. The historical knowledge might also enable learners to learn mathematics. For instance, Fauvel and van Maanen (2002) suggest that history of

mathematical concepts might highlight obstacles met in mathematical understanding. In addition, Mac an Bhaird (2009) found out that history of mathematical concepts can explain that some mathematical concepts have taken thousands of years to develop into the current form, and this could encourage the learners not to give up with their work. Mac an Bhaird (2009) further suggests that the knowledge of historical development of a concept will enable learners to see the bigger picture and realise that mathematical understanding involves success and failure.

Apart from understanding the development of the mathematical concepts, the findings have also shown that incorporating history of a mathematical concept could also enable learners to be creative in order to improve the skills, knowledge and develop new ideas in mathematics. For instance, several skills have been improved in algebra throughout its three developmental stages (Radford & Puig, 2007). Learners and teachers might be using various symbols to enhance understanding of concepts. As Malisani and Spagnolo (2008) suggested, history of mathematics might highlight the importance of symbolism in mathematics. Therefore, the creativity in mathematics could be enhanced if history of mathematical concepts is used because learners might come to realise that many mathematical ideas are hidden in the symbols or formulae used. Therefore, it can be concluded that history of a concept could spice up the learning process and aid mathematical concept understanding (Jankvist, 2011).

Furthermore, mathematics learning involves the use of different activities and as such, learners are supposed to vary their learning strategies. The study has revealed that the knowledge of the history of a mathematical concept could help learners to vary their learning strategies because learners' understanding of the history indicates that they could find what other people have been doing in mathematics and acquire required information on the possible use of different learning strategies. Therefore, use of different learning strategies would help and/or accommodate other learners to understand the concepts. Goktepe and Ozdemir (2013) argue that mathematics learning that encourage the use of the same strategies repeatedly would let learners get bored whereas involving learners to be active in the learning process would allow them to use different learning strategies, and this would increase learners' interests towards lessons (e.g. Kaphesi, 2015). It was also observed by Kjeldsen and Blomhøj (2011) that going through historical sources would allow learners to realise that many strategies of proofs for some concepts have changed over time and these might have been done so to enhance their understanding of the concepts in mathematics better. So, the history of mathematical concepts might be used to contribute to the improvement of learning of mathematics. As was also observed by Clark (2012), history enables learners to have an access to alternative ways to learn mathematics that could contribute to the understanding and utility of mathematical ideas.

Finally, the study has also found out that using history of mathematical concepts in mathematics learning might enable learners to recall concepts or methods used to solve some problems. The history of mathematical concepts acts as a term of reference when solving certain mathematics problems related to the concept as the teacher pointed. This

is so as many mathematical concepts/ideas considered nowadays were developed many years ago and are still relevant today. The knowledge of the history of mathematical concepts would help learners to deepen their understanding of mathematical ideas as learners easily recall the ideas on the topic/concept. For instance, Burns (2010) found out that historical facts or names of mathematicians could enable learners to make connections with the concepts they are learning. The learners' responses indicated that history of mathematical concept is important because they might be referring to the historical information from time to time. This might enable learners to construct appropriate meaning of the concepts they are learning. Learners' ability to recall and follow what others did in mathematics might also support their mathematical concept understanding.

4.3 History of mathematical concept and attitude towards mathematics learning

Analysis of data indicates that incorporating history of the mathematical concept in mathematics learning plays a vital role in changing learners' attitudes toward mathematics learning. It was observed during the time the teacher was introducing the topic that accompanied its historical information that learners were so interested upon hearing what had happened during the concept development and who did what. It was also observed that the learners' behaviour was positive towards using history of mathematical concept in classroom.

The analysis of learners' responses from learner questionnaire revealed that 40 of the 50 learners involved in the study perceived mathematics as any other subject. This means that they considered mathematics doable as the concepts contained in it were discovered by people like them. This implies that to some extent historical information is one of the factors that could enable them to develop positive attitude towards the mathematics. In addition, learners' responses have also indicated that they would develop a positive attitude towards learning mathematics because they initially lacked a motivating factor to learning mathematics. So, using history of mathematical concepts could motivate them as they are able to understand and know the processes involved in the concept development and this is in line with using history as a tool in teaching and learning mathematics. The results have also revealed that some learners' negative attitude towards mathematics – on how they could understand mathematical concepts – could be minimised. Some learners perceived that they could not understand mathematics because mathematics is difficult and only few learners could do better in mathematics.

In addition, it has been found out that learners would be taking mathematics lessons more seriously than before. They suggested that they would be able to come up with new mathematical ideas as they will dedicate their time to study mathematics because during the time of mathematics concept development, mathematicians were dedicated and invested a lot of time to come up with acceptable concepts.

Furthermore, the data analysis has indicated that before learners were exposed to history of mathematical concepts, they understood mathematics as a subject that only required them to deal with mathematical operations, use theorems and figures to solve problems and not on how they were developed. This limited understanding of mathematics among learners is considered as one of the factors that affect learning process and lead learners to develop negative attitude towards mathematics. Therefore, the findings from this study have indicated that this problem could be minimised if historical information of the concepts is used in the classroom because mathematics is not only about solving problems but understanding its nature or origin.

On the other hand, learners' responses indicate that they would strive more and not easily give up if they face a challenging problem. In so doing learners' knowledge would easily develop through their activeness in learning processes as is also recommended by radical constructivists. It was also observed from the responses that some learners would easily identify some concepts that could be easily understood in mathematics and find proper strategies of dealing with the difficult ones.

Some common responses from learners for this theme are given as follows:

> *I was not understanding mathematics very well because I had so many questions like who discovered mathematics or who brought solution to mathematics problems;*
> *I thought that mathematics was very tough that we are the first to do these geometry but after being exposed to history of mathematics I changed my attitude towards mathematics I can now think that if people in old days managed to do it without advanced methods that means we can also do it because we know have improved methods;*

I am being able to follow mathematics very well because I am able to know where mathematics began and is now and of course where as the world is going with mathematical ideas;

Because it encourages us to work hard because we know that other people used to pass too;

Panali povuta kumvetsa chifukwa ukamawona manambala akuphatikizidwa ndi malembo ena ndi ena zimakhala ngati za misala. (It was difficult to understand the concepts because once you see numbers being combined with other letters, it was like something being unusual).

The teacher also suggested that incorporating history of some concepts in mathematics would highlight important issues that make learners change their attitudes from negative to positive. Learners might realise that mathematical concept development is an ongoing process and different ideas are being incorporated to make it learnable and meaningful. Knowledge of the developments in the mathematics would enable learners to become motivated and strive if they want to achieve something in life that requires mathematical knowledge. History of mathematical concepts would enable learners to understand that mathematics is not static but dynamic because of the developments that have taken place so far. Learners would also be working towards analysis of different approaches of solving mathematical problems and develop their own theories and constructs. This also enables many learners to work hard in mathematics and have determined goals in life. In this case, the teacher pointed out that,

The learners will appreciate that mathematicians had problems too in discovering and proving the theorems and concepts but did not give up. Therefore, this should instill in the learners the spirit of hardworking and perseverance

Though it was observed that learners were so interested to find out how mathematical concepts came into existence and change their attitudes to liking mathematics but some limitations to using history in the classroom were also observed. (Refer to subsection **4.5** for the details of the findings on the challenges of implementing the history of mathematical concepts in the mathematics classroom.)

4.3.1 Discussion

Learning mathematics requires learners to be motivated and develop an interest towards the subject. So, it has been found out that using history of mathematical concept in learning mathematics would help learners to develop a positive attitude towards mathematics learning. The findings have shown that learners would take mathematics lessons more seriously because they know some of the processes involved in mathematical development. Learners' responses have also shown that knowing the processes on how mathematical concepts were developed would encourage them to study mathematics better and deepen their understanding of mathematics concepts (Liu, 2003; Marshall, & Rich, 2000). In this case, it might be observed that using history of mathematical concept in the classroom brings excitement in learners and make mathematics as a human activity and make mathematics lessons live and less frightening, as was also observed by Fauvel and van Maanen (2002).

On the other hand, learners' negative perceptions of mathematics, which result from bad publicity of mathematics (Mac an Bhaird, 2011) in the community they come from, could be minimised if they study the history of some concepts they are learning. Many people in the community regard mathematics as the subject that only few individuals could do because of its nature and many learners go to school with this perception. Therefore, if learners gain positive attitudes, then they will learn mathematics better because negative attitudes towards mathematics affect the learning processes (Mac an Bhaird, 2011). Charalambous et al. (2008) also found out that sometimes teachers' attitudes towards mathematics also affect learners' attitudes towards mathematics.

In addition, as learners' responses have shown, many had questions regarding the mathematics concepts, how they were developed, who played part and their origin. So, the knowledge they would acquire after incorporating history of mathematical concepts could play a crucial role in the learners' attitudes towards learning mathematics. According to the constructivists views on mathematics learning, learners construct their own knowledge of concepts if they actively take part in the whole learning process (Yackel et al., 2011). Therefore, it might be concluded that historical knowledge of mathematical ideas would enhance learners understanding of mathematical ideas as they might be able to actively search for the roots and the development of some concepts on their own. These findings are also consistent with those of Charalambous et al. (2008) and McBride and Rollins (1977) about the effects of history of mathematics on the learners' attitudes towards mathematics because history has the ability to improve positive attitudes of learners towards mathematics. Although some benefits of history of

mathematics are indirect to the learners' performance in mathematics since it lacks assessment (Panasuk & Horton, 2013), the action taken after the knowledge acquired from history could enable learners to change their attitudes towards mathematics learning as well as their motivation to learn more in mathematics.

4.4 Incorporating history of mathematical concept in mathematics classroom

The findings presented in this subsection relates to the third research question of the study – concerning when and how to incorporate the history of the mathematical concept in the mathematics classroom. Data obtained from three different instruments have been analysed and findings are presented below.

In the classroom, the researcher observed that incorporating history on how the topic was developed and the development of some concepts in the classroom covered few minutes. For instance, the historical information the teacher told the learners in the classroom took about five (5) minutes. The teacher was able to mention the names of all mathematicians that were involved in the development of the topic, the century in which the topic was discovered by these mathematicians and how some concepts were developed within the topic. This means that teaching and learning mathematical concepts through its history does not necessarily require additional time. The allocated time on the topic is enough for the inclusion of historical information in the classroom.

The summary of results of the classroom observations are displayed in table 1.

Table 1: Lesson observations on incorporation of history in mathematics classroom.

Lessons observed	**Observations**	**Not observed**
LESSON ONE Concepts discussed in lesson one were about Cartesian plane, locating points on the Cartesian plane, coordinates and length of a line segment	The teacher introduced the mathematics topic by outlining the historical background of the topic as given in **section 2.2.** This was done before the learners were taught the actual concepts from the topic. It was also observed that within the lesson the teacher briefly talked about the history behind the Pythagoras's theorem because the theorem was used to calculate the length of the line segment	

Lessons observed	Observations	Not observed
LESSON TWO The midpoint of a line segment and the gradient concepts were discussed	The historical background of the gradient/slope/tangent concept was observed to have been taught together with the actual content. The teacher talked about the how Egyptians used the concept and other mathematicians that have developed the concept further	
LESSON THREE The linear equation concept was discussed	The ideas of using letters from the English alphabet for known quantities (constants) and unknown quantities (variables) were ably taught together with the concept	However, it was observed that the history of the linear equation was not taught simultaneously with linear equation concept.

Lessons observed	Observations	Not observed
LESSON FOUR The lesson was about graphs and parallel lines		It was also observed that historical information of these concepts was not outlined in the classroom when teaching them.

Apart from the findings from the lessons observed, it was also discovered from the teacher questionnaire that history of the mathematical concept could be incorporated during the introduction of the topic. It was found out from the teacher that it is necessary to decide when and how history should be used in the classrooms. This could be done at any point in the classroom either when introducing a new topic in classroom and outlining the general information of how the topic was developed or giving learners reading assignments to find the information on their own. The general information could be about the mathematicians' biography, centuries in which some mathematical concepts were developed, and some interesting stories behind the development of the concepts to be discussed. When the teacher was asked the following question: **when and how can you incorporate history of mathematics in mathematics classroom?** He suggested that, *in the introduction of the topic, teachers should explain to the learners some of the people who contributed to easing of solving the concerned topic or concepts.*

Learners were also asked during the focus group discussion about how and when they prefer to be told the historical information of the concepts. It was observed from the interview that learners preferred to be told the history of a mathematical concept every time the teacher introduces new concepts in class. Learners pointed out that they would be interested to know how the concepts were developed, what skills were used and which ideas they can learn from them. Learners also agreed with the teacher that history of a mathematical concept in a topic could be highlighted during its introduction and within the topic. Learners suggested that the teacher should be prepared to incorporate the historical information in the lessons.

Here is an excerpt of the interview the researcher had with selected learners on the possible inclusion of history of mathematical concept in mathematics classroom. Learners have been presented as Learner **A**, **B**, **C** and so on.

Researcher: Now you want the teacher to be telling you something on the history of mathematical concept, so how should he/she be doing this? Do you want the teacher to be telling you this anytime he or she comes to class or there should be time for that purpose or what do you think?

Learner A: Iwowo akamayamba, akamakoza kuti topic ikubwerayi mesa iyeyu ndi mphunzitsi azipanga study. Akumayenera kupangiratu kumenekoko chifukwa poyamba (inaudible) pazikhala pachiyambi pa topic yakeyo azikatiuza mclass. (The teacher decides the topic to

teach, so before coming to class to teach he or she should find its history and what exactly to tell the learners from history. This should be done at the beginning of the topic)

Researcher: What should the teacher do if the topic has so many different small units that other people also did and the teacher cannot mention all of them at the same time?

Learner B: Pamenepopo pali masiku ena pamakhala kuti pali two periods ndiye akhoza kugawa kuti period yoyamba apange ma areawo inayo apange zinazo. (If it is like that then there are other days when the teacher has two periods, so he or she can use one period for talking about history of mathematical concept and the other period for teaching actual content)

Learner C: Pamenepopo poti pali ma areas areas ndiye kuti area ina iliyonse akamailowa azikhala kuti history yake watengeratu chire. Ina iliyonse history yake watengeratu chire kuti zisamatisokoneze. (Where there are so many small areas, then the teacher should consider the history of each area separately when teaching so that this does not interfere with the learning process)

4.4.1 Discussion

In the Malawian context, history of some mathematical concepts is not clearly presented in the syllabus as such mathematics teaching and learning does not use its history. This is why this study aimed at exploring how and when the history of some concepts could be used in mathematics learning. The study has observed that history of mathematical concepts could be incorporated in different ways, either at the introduction of the topic or every time a new concept is introduced in the classroom. As was also observed by Burns (2010), history of mathematical concepts could be used as an introduction of interesting extra knowledge. Presenting the historical information of the concepts at different stages in the lessons could serve different purposes. For example, Siu (2000) suggested that this can provide "motivation and perspective so that learners know what they are heading for or what they have covered, and how that relates to knowledge previously gained" (p. 5). It can also help to aid the understanding of the concepts if history is used as a tool in teaching and learning as Jankvist (2009) argued.

The approach of incorporating history of mathematical concept considered in this study is only one of several possible ways of integrating history into the classroom or curriculum. Since concepts in mathematics were developed by different people, the complexity of these concepts is also different, and this is also observed in how they are presented in the curriculum. So, different approaches could be used to let learners understand the history and find the relevant information. The weakness of this approach in the study is that learners could be passively receiving information which they could not consider relevant to their learning process. However, this approach is suitable in the situations where some

materials are not readily available for reference and for the introduction of new concepts and ideas in mathematics. The information acquired through this approach could still give an insight into some learners to start searching for the history of other concepts in mathematics that later could help them to understand the concepts better. According to Burns (2010), when learners are introduced to a new topic or concept, they should also be introduced to the history of that topic. This is done with the aim of enlightening learners about the concept and tries to motivate them to construct their own knowledge from it.

Since the history of mathematical concepts could not always be used in the introduction of the lesson or topic, it could still be used where it is needed anywhere within the topic or lesson. Therefore, teachers being facilitators in the mathematics classroom (Thompson, 2000) are supposed to decide what to select from history of mathematics and when to use the selected information so that this might enable their learners to develop proper skills needed in knowledge construction. Hence, this information could help learners to understand that mathematicians learnt from themselves, they constructed knowledge themselves, and they shared it with others for discussion – as sharing knowledge is important in one's mental development.

4.5 Challenges of implementing the history of mathematical concepts in mathematics classroom

Despite discussing with the teacher the history of some concepts attached to the topic to be presented in class, the information was not used in the other lessons observed as shown in table 1 in subsection 4.4. This was because it was suggested that this could

confuse learners. For instance, the teacher felt that if he uses all the information regarding the development of a concept, then this has a possibility that it might "confuse learners by taking what is being said as important not the mathematics" (Teacher's belief from the interview). He further argued that, "if I continue telling them about history of mathematical concepts, they might forget to capture the intended objectives of the lesson. They might take the history of mathematics as the mathematics to learn". On the other hand, it was also observed that learners might consider the historical information as very important and fail to understand the required concepts. For instance, in the questionnaire, the teacher pointed out that,

> *Some learners may think that the history of mathematics is among the specific objectives to be examined during the examinations hence compromise the intended outcome from the syllabus.*

This also means that telling learners the developmental stories of mathematical concepts they are learning could make learners concentrate on these stories and not on the actual content that is examinable and in the end fail to achieve the intended objectives.

The other challenge that was observed from both the teacher and the learners was, lack of time. The teacher suggested that he might be spending a reasonable amount of time on the teaching of history instead of using that time on the actual content that is examinable. On the other hand, some learners during the interview argued that including history of mathematical concept in mathematics classroom might not allow them to learn a lot of mathematics because the other time might be spent on historical stories and not on the mathematical content.

For instance, some of the learners' responses from the interview are presented as follows:

Learner D: Komanso ndikuwona ngati zizidya nthawi azikhala ndi tsiku loti period imodzi azikamba zoti history ka nthawi kazikhala ngati kakudyeka limodzi ndi zimene amafuna atiphunzitse tsiku limenelo. (But I think this will be using our time a lot. The teacher should have a special day for talking about history of mathematics because this will be consuming time for the day's lesson if the teacher combines these two).

Learner E: Inde akamapanga daily inde tizikumbukira koma sitimaphunzira masamu ambiri. (If this is done on daily basis, yes we might be able to remember a lot, however, we will not be learning a lot of mathematics).

On the other hand, it was observed that some learners' perception towards mathematics is that mathematics is a subject that deals with calculations not involving too much reading as is involved in other subjects. Therefore, learning mathematics by using some aspects of history of those concepts could affect their learning process negatively. For instance, one of the learners suggested that "*it requires reading which is not needed in mathematics".* This is one of the perceptions learners have on mathematics that they are supposed to solve mathematical problems only and not to involve much in reading the texts of mathematics. So, instead of solving the mathematical problems, they will be searching for the documents that contain the historical information. It has also been found out that mathematics teaching and learning has been much focused on getting right

numerical answers and not on analysing situations to acquire other skills from ancient mathematicians. So they consider history of mathematical concept as not part of real mathematics. For instance, one of the learners suggested that *"it is not fair because if it is history should be history not mathematics".*

It has also been found out that lack of relevant mathematics materials from the history would affect how they can learn mathematics through its history. For instance, learners suggested that there are no teachers to handle the subject, and there are no relevant mathematics textbooks on the history of mathematics they could refer to at secondary school level. In addition, some learners might not be flexible in doing mathematics as they might stick to the historical way of doing things.

Learners' common responses for this theme are given as follows:

> *It requires reading which is not needed in mathematics. You cannot use your own way but you use the way that is history of mathematics says;*
> *Shortage of mathematical materials;*
> *Lack of mathematics equipment;*
> *It is a lot bit confusing because we learn about past, present so we get a little bit confused but anyway its interesting (sic).*

4.5.1 Summary

There are some challenges pertaining to the implementation of history of mathematical concepts in the classroom. The study has identified some of them in the Malawian context such as: some learners might be confused because they are supposed to learn two different issues at the same time, which is history and the mathematics itself. Using history in classroom instructions would affect the coverage of mathematical content as

some time might be used for that purpose. The study has also indicated that learners might be focusing much on the historical information and not on the actual content that is required for examination and later affect their performance during the examinations. On the other hand, it has been observed that learners hate too much reading in mathematics, they consider using history in classroom would allow them to read a lot. The study has also observed that there is lack of relevant resources to be used by the learners.

4.5.2 Discussion

The findings have shown that some learners might be confused because they are supposed to learn two different issues at the same time: history and mathematics. This confusion might not enhance understanding of mathematical concepts, but some scholars have observed that this might help learners understand the concepts better. For instance, Burns (2010) observed that the use of history might improve learners' own knowledge, because they might be able to trace the paths through which mathematics was developed. Similarly, Panasuk and Horton (2013) assert that using history in the classroom instruction captures learners' interests as such they might improve their attitudes toward mathematics which in turn could play a critical role in mathematics knowledge construction. So, learning mathematics does not only require learners to achieve computational skills but other skills as well, and, through the use of history, learners could also achieve other skills such as reasoning.

The findings of the study also indicate that there is limited time to implement it in the classroom. Some learners pointed out that they might not be learning a lot of mathematics. However, with proper planning, this could not affect the allocated time to the subject. For instance, it has been observed in the studies Clark (2012) and Haverhals and Roscoe (2010) conducted that the same time allocated to the course could be used and no special time could be sought. In addition, some learners suggested that a special day could be set for history of some mathematical concepts whilst others did not agree with that idea. The drawback of having a day to talk about history is that learners might not be able to link the concepts well with its history. Therefore, this might be taken as support for incorporating history in regular mathematics lessons. The observations from the teacher and the learners indicate one of the weaknesses of the approach considered in the study as it relied on the teacher to highlight the historical information in the classroom. However, by using other ways of presenting history to learners, such as the module approach, this problem of lack of time to use history in the classroom might be taken care of because learners might be able to refer to the documents on their own (Jankvist, 2011). On the other hand, radical constructivist views suggest that knowledge construction is an individual's act, therefore, other learners could be motivated with the information and eager to find out more on the history whilst others could not be motivated hence affecting their learning process. The motivated learners could do a lot of research in the subject so that they find the relevant origin of the concepts after getting an insight into the history of a concept in mathematics.

It has also been observed in this study that learners have different perceptions on mathematics that also affect how they understand some mathematical concepts. Learners develop negative attitudes towards mathematics from different situations, for instance, some learners do not like history subjects. These findings concur with the study Charalambous et al. (2008), who observed that some learners hate history subjects. Using history to learn mathematics may thus lead them to develop negative attitude towards mathematics. However, Haverhals and Roscoe (2010) found out that learners enjoy the lessons that are supplemented with historical information which is also contrary to what Siu (2007) observed that using history makes mathematics lessons boring.

It has also been noted that although history of mathematics is not yet incorporated into the Malawian mathematics curriculum, learners had an observation on the lack of relevant resources required in its study. Therefore, one might understand that these learners are aware of the Malawian educational system, because throughout their time in schools they have not learnt mathematics through its history. Many authors have hardly included history of the concepts in the learners' mathematics textbooks and learners could not easily find the information on the development of mathematical ideas. Though there are different ways of getting the historical information, many of these ways are not accessible to most secondary school learners, therefore, affecting how they can learn or get the required information for doing some reading assignments.

4.6 Summary of the chapter

This chapter outlined the results of the study and the discussion of the findings. It has been found out that there are many benefits of incorporating history of some mathematics concepts than the challenges of implementing the history. The findings of the study were categorised based on the research questions. Therefore, the research questions acted as the headings for different section. The chapter to follow considers the conclusion of the research findings, their implications and the recommendations and the area for further study.

CHAPTER FIVE

CONCLUSIONS AND IMPLICATIONS

5.1 Conclusions

The study had four research questions. The first research question was about exploring some possible benefits of integrating history of mathematical concepts in developing the understanding of mathematical concepts in learners. The second research question was about finding some effects of using history of mathematical concepts on learners' attitudes towards mathematics. The third research question sought to explore how and when the history of mathematical concept might be incorporated and the fourth research question was about challenges learners could face if it is incorporated into the mathematics classroom in upper secondary school level in Malawi.

The study has found out that there are many possible benefits of incorporating history of mathematical concept into mathematics learning in the Malawian context. It has been revealed that learners would acquire the general knowledge on how the mathematical ideas were developed, and this is using history as a goal in teaching and learning mathematics and this knowledge would also enhance their reasoning ability in doing mathematics. So, learners' understanding of mathematical ideas is believed to improve through the use of history of some mathematical concept in learning mathematics.

It has also been observed that learners would want to learn mathematics with the inclusion of some historical information, because the historical knowledge would enhance their creativity as ancient mathematicians were so creative, which is referred to using history as a tool in teaching and learning mathematics. In this case, it can be concluded that history of any mathematical concept could act as a catalyst to the mathematics learning process, hence aiding understanding of the mathematical concepts.

Integrating history into the mathematics classroom also tend to enable learners to vary their learning strategies to improve their understanding in the study of mathematics and this is one of the importance of using history as a tool in the classroom. The sentiments from both the teacher and learners consider the use of history of mathematical concepts as a guide to recall some concepts as they are doing mathematics that enhance understanding of mathematical concepts and knowledge construction.

The findings have also indicated that using history to learn mathematics would enable learners to remember easily the concepts they have learnt or refer to the concepts from the original sources if they get stuck or some ideas are not clear to them. Learners could also discover other mathematical ideas that are not presented in the new concepts they are learning.

In addition, mathematics learning can be enhanced if learners develop a positive interest towards the subject. The study has revealed that, apart from benefiting from historical information on knowledge construction and understanding, some learners developed

positive attitude towards mathematics. This is so as some of the questions they had concerning the development of some mathematical concepts could be answered if they also study the history of the subject. So, this study provided them with an insight of the nature of mathematics. Therefore, it can be concluded that there is a positive effect in using history as a tool and history as a goal in teaching and learning mathematics. Many learners' motivation is increased towards learning mathematics (history as a tool) and history enables them to understand the nature of mathematics (history as a goal). The study also motivated learners to look for the historical information for some of the concepts they are learning, so that they find out other historical information of the concepts on their own.

Furthermore, the study has found out that history of the concept could be incorporated in the lessons as one of the points to highlight the concepts so that learners' understanding could be improved though the Malawian mathematics curriculum does not suggest the inclusion of it. As there are many ways of incorporating history into the lessons, the study noted the basic considerations on the inclusion of history. For instance, employing illumination approaches that enable teachers to use names of mathematicians, the dates on which the concepts were developed, some interesting backgrounds on some of the concept developed and any other relevant historical information.

The study also wanted to find out the challenges in the implementation of history in the classroom. So, the study found out some challenges on both the implementer (the teacher) and the learners. It was observed that some learners might be confused if history

of the concept is combined with the content they are supposed to learn, and that would affect how they could understand the concepts as some learners pointed out that they might be required to study a lot to understand the history and the mathematics. Learners also pointed out that they might not be learning a lot of mathematics, as the other time might be used to state the history. On the other hand, learners who do not like history as a subject might also not like the historical information of the concept. The other challenge is on the availability of resources which are not readily available.

5.2 Implications of the findings

The study has some implications for mathematics teaching and learning considering the way learners construct knowledge and how that knowledge construction might be enhanced through the use of history of the concepts.

Teaching and learning processes might be enhanced if learners understand how some of the concepts exist. As has been found out, the use of historical information motivates learners to study mathematics and some learners could find possible answers to the questions they had concerning the development of mathematical concepts. This implies that learners might be willing to learn and investigate in order to understand the mathematical concepts for themselves and make the teaching and learning processes easier. Learners' knowledge construction might be enhanced as history might facilitate the study of mathematics. As learners investigate the history of some concepts, they might find appropriate strategies that would enhance their knowledge construction. In addition, learners' creativity improves as they link different concepts to come up with

new concepts or find other methods of solving problems. This also means that learners' minds are provoked to think critically and increase their conceptual understanding as they learn from what others did in mathematics. Learners, sometimes learn by imitation, therefore, history of mathematical concepts could enable learners to borrow the ideas ancient mathematicians were using to construct the mathematical knowledge or ideas/concept.

On the other hand, incorporation of history in mathematics classroom does not require teachers to be historians of mathematics as only essential information is included in the lessons. Therefore, teaching and learning of mathematical content might not be affected.

Since mathematics is often taught as a finished product, the use of history might enlighten learners that mathematical ideas are continuously being developed and improved. Learners might also be able to trace the paths through which a mathematical concept is developed until the present day form. The activities that learners might be doing could encourage their participation in learning mathematics.

The challenges learners could face in the classroom if history is used, and in searching for the origin sources of some concepts, could help teachers support them in the learning process. Therefore, if history is not used in teaching and learning mathematics, many learners will still lack motivation that will affect how they learn and understand many mathematical concepts.

5.3 Area for further research

Other studies could be conducted in the same field, for instance, focusing on the mathematics textbooks by finding relevant topics and areas in which the history could be used, a qualitative or quantitative study could be done on teachers to find their attitudes toward using history in teaching secondary mathematics.

REFERENCES

Anderson, J. R., Reder, L. M., & Simon, H. A. (1998). Radical constructivism and cognitive psychology. In D. Ravitch (Ed.), *Brookings papers on education policy 1998* (pp. 227–278). Washington, DC: Brookings Institute.

Avital, S. (1995). History of mathematics can help improve, instruction and learning. In Swerts, F., Fauvel, F., Bekken, O., Johansson, B. & Katz, V. (Eds.), *Learn From The Masters* (pp. 1–12). The Mathematical Association of America.

Bagni, G.T. (2000). The role of the history of mathematics in mathematics education: Reflections and examples. In I. Schwank (Ed.), *Proceedings of the First Conference of the European Society for Research in Mathematics Education, Vol. II* (pp. 220–231). Osnabrück, Germany: Forschungsinstitut für Mathematikdidaktik.

Barnett, J.H., Lodder, J., & Pengelley, D. (2013). The pedagogy of primary historical sources in mathematics: Classroom practice meets theoretical frameworks. *Science & Education, 1*, 1–21.

Boyer, C.B. (1968). *A history of mathematics*. John Wiley and Son.

Burns, B.A. (2010). Pre–service teachers' exposure to using the history of mathematics to enhance their teaching of high school mathematics. *IUMPST: The Journal, 4* (Curriculum).

Burton, D.M. (2011). *The history of mathematics: An introduction* (7th ed.). New York, NY: Mc Graw Hill.

Bütüner, S. Ö. (2015). Impact of using history of mathematics on students' mathematics attitudes: A meta–analysis study. *European Journal of Science and Mathematics Education, 3*(4), 337–347.

Charalambous, C.Y., Panaoura, A., & Philippou, G. (2008). Using the history of mathematics to induce changes in pre–service teacher's beliefs and attitudes: Insights from evaluating a teacher education program. *Educational Studies in Mathematics, 71*, 161–180.

Clark, K.M. (2012). History of mathematics: illuminating understanding of school mathematics concepts for prospective mathematics teachers. *Educational Studies in Mathematics 81*, 67–84.

Clark-Carter, D. (2004). *Quantitative psychological research: A student's handbook*. Psychological Press.

Cohen, L., Mannion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). London: Routledge Falmer.

Creswell, J.W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Creswell, J.W. (2012). *Educational research. Planning, conducting and evaluating quantitative and qualitative Research* (4th ed.). Boston, MA: Pearson Education.

Farmaki, V., & Paschos, T. (2007). Employing genetic 'moments' in the history of mathematics in classroom activities. *Educational Studies in Mathematics 66*, 83–106.

Fauvel, J. (1991). Using History in Mathematics Education. *For the learning of Mathematics*, 11(2), 3–6.

Fauvel, J., & van Maanen, J. (Eds.) (2002). *History in mathematics education: New ICMI study*. Dordrecht: Kluwer Academic.

Flyvbjerg, B. (2006). Five misunderstandings about case–study research. *Qualitative inquiry, 12*(2), 219–245.

Fried, M.N. (2007). Didactics and history of mathematics: Knowledge and self-knowledge. *Educational Studies in Mathematics, 66*(2), 203–223.

Glasersfeld, E. V. (1989). Constructivism in education. In Husen, T. & Postlethwaite, T. N. (eds.) (1989). *The International Encyclopedia of Education, Supplement, 1,* Oxford/New York: Pergamon Press, 162–163.

Glasersfeld, E.V. (1995). *Radical constructivism: A way of knowing and learning*, Studies in mathematics education. London: Falmer Press.

Glatthorn, A.A., & Joyner, R.L. (2005). *Writing the winning thesis or dissertation: A step–by–step guide* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Goktepe, S., & Ozdemir, A. S. (2013). An example of using history of mathematics in classes. *European Journal of Science and Mathematics Education, 1*(3), 125–136.

Grugnett, L. (2000). The history of mathematics and its influence on pedagogical problems. In V. J. Katz (Ed.), *Using History to Teach Mathematics. An International Perspective* (pp. 29–35). Washington, DC: The Mathematics Association of America.

Haverhals, N., & Roscoe, M. (2010). The history of mathematics as a pedagogical tool: Teaching the integral of the secant via mercator's projection. *The Montana Mathematics Enthusiast, 7*(2&3), 339–368.

Horton, L., & Panasuk, R.M. (2011). Raising awareness the history of mathematics in high school curriculum. *International Journal of Humanities and Social Science, 1*(16), 37–46.

Jankvist, U.T. (2009). A categorization of the “whys” and “hows” of using history in mathematics education. *Educational Studies in Mathematics, 71,* 235–261.

Jankvist, U.T. (2010). An empirical study of using history as a ‘goal’. *Educational Studies in Mathematics, 74*(1), 53–74.

Jankvist, U.T. (2011). Anchoring students’ metaperspective discussions of history in mathematics. *Journal for Research in Mathematics Education, 42*(4), 346–385.

Jankvist, U.T., Mosvold. R., Fauskanger, J., & Jakobsen, A (2015). Analysing the use of history of mathematics through MKT. *International Journal of Mathematical Education in Science and Technology, 46*(4), 495–507.

Jankvist, U.T., & Kjeldsen, T.H. (2010). New avenues for history in mathematics education: Mathematical competencies and anchoring. *Science and Education, 20,* 831–862.

Kaphesi, E. (2015). A study of mathematics learning strategies among third year secondary school students in Malawi. *Malawi Journal of Education and Development, 5,* 1–18.

Katz, V., & Barton, B. (2007). Stages in the history of algebra with implications for teaching. *Educational Studies in Mathematics, 66,* 185–201. Special issue on the history of mathematics in mathematics education.

Katz, V.J. (2009). *A history of mathematics: An introduction* (3th ed.). Boston, MA: Pearson Education.

Kjeldsen, T.H., & Blomhøj, M. (2011). Beyond motivation: History as a method for learning meta – discursive rules in mathematics. *Educational Studies in Mathematics, 80*(3), 327–349.

Liu, P.H. (2003). Do teachers need to incorporate the history of mathematics in their teaching? *The Mathematics Teacher, 96*(6), 416–421.

Liu, C.H., & Matthews, R. (2005). Vygotsky's philosophy: Constructivism and its criticisms examined. *International Educational Journal, 6*(3), 386–399.

Mac an Bhaird, C. (2009). Introducing the history of mathematics to third level students with weak mathematics background: A case study. *Proceedings of the 2008 CELT–MSOR Conference*. Retrieved, September 23 2016 from http://eprints.teachingandlearning.ie/2804/1/Mac%20an%20Bhaird%202008%20CM–History–of–Mathematics.pdf.

Mac an Bhaird, C. (2011). The provision of mathematics support and the role of the history of mathematics. *Hermathena,* 191, 53–67.

Macmillan English Dictionary for Advanced Learners, (2002). Macmillan Publishers Limited.

Malisani, E., & Spagnolo, F. (2008). From arithmetic thought to algebraic thought: The role of the "variable". *Educational Studies in Mathematics, 71*, 19–41.

Mapotse, T.A., & Gumbo, M.T. (2015). Emancipating secondary school teachers from their technology and pedagogical challenges: An action research study. In *Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE)*. Proceeding of the 23rd annual meeting on "Mathematics, science and technology education for empowerment and equity". Mozambique, Pedagogical University and Eduardo Mondlane University, Maputo, 13 –16 January.

Marshall, G.L., & Rich, B.S. (2000). The role of history in a mathematics class. *The Mathematics Teacher, 93*(8), 704–706.

McBride, C.C., & Rollins, J.H. (1977). The effects of history of mathematics on attitudes towards mathematics of college algebra students. *Journal for Research in Mathematics Education, 8*(1), 57–61.

Merzbach, U.C., & Boyer, C.B. (2011). *A history of mathematics* (3rd ed.). Hoboken, NJ: John Wiley and sons.

Malawi Institute of Education (2013). *Malawi junior secondary teaching syllabus for Mathematics: (Forms 1 and 2)*. Domasi: Malawi Institute of Education.

Mosvold, R. (2006). *Mathematics in everyday life: A study of beliefs and actions* (Doctoral dissertation). University of Bergen, Bergen, Norway.

Moyo, I. (2014). *An investigation into the factors impacting on the selection and adoption of constructivist teaching methods by mathematics teachers in selected Gauteng urban schools (*Master's Thesis). University of South Africa, South Africa.

Ndengu, D.M. (2012). *Describing and conducting qualitative research: A guide for postgraduate students in the social sciences*. Mzuzu: MZUNI Press.

Onwuegbuzie, A.L., Leech, N.L., & Collins, K.M.T. (2010). Innovative data collection strategies in qualitative research. *The Qualitative Report, 15*(3), 696–726.

Ozdemir, A.S., Goktepe, S., & Kepceoglu, (2012). Using mathematics history to strengthen geometric proof skills. *Prodedia-Social and Behavioural Sciences, 46*, 1177–1181.

Panasuk, R.M., & Horton, L.B. (2013). Integrating history of mathematics into the classroom: Was Aristotle wrong? *Journal of Curriculum and Teaching*, 2(2), 37–46.

Panasuk, R.M., & Horton, L.B. (2013). Integrating history of mathematics into the curriculum: What are the chances and constraints? *International Electronic Journal of Mathematics educators,* 7(1), 3–20.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: Information Age Publishing.

Radford, L., & Puig, L (2006). Syntax and meaning as sensuous, visual, historical forms of algebraic thinking. *Educational Studies in Mathematics, 66,* 145–164.

Siu, M.K. (2000). The ABCD of using history of mathematics in the (undergraduate) classroom. In V. J. Katz (Ed.), *Using History to Teach Mathematics: An International Perspective* (pp. 3–9). Washington, DC: The Mathematics Association of America.

Siu, M.K. (2007). No, I don't use history of mathematics in my class. Why? In F. Furinghetti, S. Kaijser, & C. Tzanakis (Eds.), *Proceedings HPM2004 & ESU4* (revised edition, pp. 268–277). Uppsala: Uppsala Universitet.

Stillwell, J. (2010). *Mathematics and its history* (3rd ed.). San Francisco, CA: Springer.

Stuart, J., Akyeampong, K., & Croft, A. (2009). *Key issues in teacher education*. Oxford: MacMillan Publishers.

Thomaidis, Y., & Tsanakis, C. (2007). The notion of historical "parallelism" revisited: Historical evolution and student's conception of the order relation on the number line. *Educational Studies in Mathematics, 66,* 165–183.

Thompson, P.W. (2000). Radical constructivism: Reflections and directions. In L.P. Steffe & P.W. Thompson (Eds.), *Radical constructivism in action: Building on the pioneering work of Ernest von Glasersfeld* (pp. 422–448). London: Routledge.

Tzanakis, C., & Arcavi, A. (2000). Integrating history of mathematics in the classroom: An analytic survey. In J. Fauvel, & J. van Maanen (Eds.), *History in mathematics education* (pp. 201–240, Chapter 7). The ICMI Study. Dordrecht: Kluwer Academic Publishers.

Yackel, E., Gravemeijer, K., & Sfard, A. (Eds.) (2011). *A journey in mathematics education research: Insights from the work of Paul Cobb*. Dordrecht: Springer.

Yee, L.S., & Chapman, E. (2010). Using history to enhance students learning and attitudes in Singapore mathematics classrooms. *Education Research and Perspectives, 37*(2), 110–132.

APPENDICES

Appendix A: Letter of Introduction from Chancellor College

PRINCIPAL
Richard Tambulasi, B.A (Pub Admin), BPA (Hons), MPA, Ph.D

CHANCELLOR COLLEGE
P.O. Box 280, Zomba, Malawi
Telephone: (265) 524 222
Fax: (265) 524 046
E-mail: principal@cc.ac.mw

OFFICE OF THE DEAN OF EDUCATION

18 November 2015

TO WHOM IT MAY CONCERN

Dear Madam/Sir,

RE: MR COSMAS KATHUMBA

This is to confirm that Mr Cosmas Kathumba is a Masters student under Faculty of Education at Chancellor College, University of Malawi. Mr Kathumba is required to conduct Mathematics Education research in Malawi secondary schools as part of his Masters research study. Please assist him accordingly.

If you have any questions or queries about Mr Kathumba or his research study, please feel free to contact me or the Dean of Education.

Your assistance will be greatly appreciated

UNIVERSITY OF MALAWI
CHANCELLOR COLLEGE
2015 -11- 18
DEAN
FACULTY OF EDUCATION

Dr Mercy Kazima Kishindo
Coordinator of Mathematics Education Masters programme
Email: mkazima@cc.ac.mw,
Phone: 0111955757 (office)

Appendix B: Teacher's Questionnaire

Dear Mathematics teacher,

REQUEST TO COMPLETE A RESEARCH QUESTIONNAIRE

I, Cosmas Kathumba, am a Master of Education student in Curriculum and Teaching Studies in Mathematics and Science Education at the University of Malawi, Chancellor College (Faculty of Education). Am currently conducting research for my thesis on ***the role of history of mathematical concepts in learning mathematics.***

This questionnaire seeks to collect qualitative data, therefore, you are requested to respond to the questions given. Write your responses in the spaces provided. There are no right or wrong responses and your responses will be treated with the utmost confidentiality. The information provided in this questionnaire is for research purposes only and not otherwise.

I would like to thank you in advance for your willingness to take part in this study.

Kind Regards

Yours

Cosmas J. Kathumba (MED-MAT-04-14)

---

1. If history of mathematical concepts is incorporated in the classroom, what could be the benefits of using it in learners on learning mathematics

   _____

   _____

   _____

   _____

2. What are some of the reasons that might indicate that learners like the lessons that used history of mathematical concepts?

_____

_____

_____

_____

3. How can history of mathematical concepts be incorporated in mathematics classroom?

_____

_____

_____

_____

4. What could be the challenges of using history of mathematical concepts in teaching and learning?

_____

_____

_____

_____

5. Why Malawian mathematics curriculum does not include history of mathematical concepts?

_____

_____

_____

_____

6. Suggest the ways in which the use of history of mathematical concepts in teaching help in dealing with the challenges learners face in learning mathematics.

_____

_____

_____

_____

7. How can the knowledge on how great mathematicians struggled in developing the concepts help the mathematics learning process?

   _____

   _____

   _____

   _____

8. How can the knowledge of history of mathematical concepts help you change your teaching styles?

   _____

   _____

   _____

   _____

9. Which teaching strategies could be used in the lessons that incorporate history of mathematical concepts?

   _____

   _____

   _____

   _____

10. How can you evaluate the lessons that incorporated history of mathematical concepts?

    _____

    _____

    _____

    _____

Thank you very much for your contributions!!!!

Appendix C: Learners' Questionnaire

Dear Respondent,

REQUEST TO COMPLETE A RESEARCH QUESTIONNAIRE

I, Cosmas Kathumba, am a Master of Education student in Curriculum and Teaching Studies in Mathematics and Science Education at the University of Malawi, Chancellor College (Faculty of Education). Am currently conducting research for my thesis on ***the role of history of mathematical concepts in learning mathematics.***

This questionnaire seeks to collect qualitative data, therefore, you are requested to respond to the questions given. You may respond using the language you are comfortable with (i.e Chichewa or English). Write your responses in the spaces provided. There are no right or wrong responses and your responses will be treated with the utmost confidentiality. The information provided in this questionnaire is for research purposes only and not otherwise.

I would like to thank you in advance for your willingness to take part in this study.

Kind regards

Cosmas J. Kathumba (MED-MAT-04-14)

Your teacher when introducing the topic "Coordinate Geometry" and teaching of other concepts talked about the people who were involved, the processes and the centuries in which these mathematical concepts were discovered.

1. Why the topic of coordinate geometry is also called Cartesian Geometry

   _____

   _____

   _____

   _____

2. How did that information help you approach mathematics?

   _____

   _____

   _____

   _____

3. How has this brief history of coordinate geometry helped you learn mathematics?

   _____

   _____

   _____

   _____

4. How have you benefited from the lesson that used history of mathematical concepts?

   _____

   _____

   _____

   _____

5. Why is it important to know what other people did in mathematics we are learning today?

   _____

   _____

_____

_____

6. To what extend has the use of history of mathematical concepts in learning mathematics helped you in understanding the mathematical concepts?

_____

_____

_____

7. How has the use of history of mathematical concepts changed your reasoning ability in solving problems in mathematics?

_____

_____

_____

8. How has the use of history of mathematical concept helped you change your view of mathematics?

_____

_____

_____

9. What was your understanding of mathematics before being exposed to history of mathematical concepts?

_____

_____

_____

10. What have you learnt from the use of history of mathematical concepts in your classroom?

_____

_____

_____

11. What do you think about learning mathematics in future through its history?

    _____

    _____

    _____

12. It took so many years for these mathematical concepts to be developed and other people have also modified them to suit the present day. How does this knowledge aid your reasoning ability in mathematics?

    _____

    _____

    _____

13. What could be the importance of knowing some of the problems or challenges the ancient mathematicians faced when discovering the concepts?

    _____

    _____

    _____

    _____

14. List down the challenges you have faced in the lessons that used history of mathematical concepts.

    _____

    _____

    _____

THANK YOU VERY MUCH FOR YOUR PARTICIPATION!!!!

Appendix D: Classroom Observation Checklist

Activity	Observed	Not observed
Learners' and teacher's behaviour on the use of history of mathematical concepts in the classroom		
Difficulties the teacher could face during the time of incorporating history in the classroom		
Instructional strategies used to incorporate history in the mathematics classroom		
Frequency of incorporating history in the mathematics classroom		

Appendix E: Learners' Interview Guide

INCORPORATING AND THE CHALLENGES OF IMPLEMENTING HISTORY OF MATHEMATICAL CONCEPTS IN THE CLASSROOM

1. When should the teacher tell you about the history of some mathematical concepts?
2. How often should the teacher be telling you the history of mathematical concept?
3. How should the teacher use the history of some mathematical concepts in mathematics classroom?
4. How can the use of history of mathematical concepts in the classroom affect your learning process?

Appendix F: Teacher' Interview Guide

1. How much information would you use from history in the classroom?
2. What challenges would you face if you want to implement history in the classroom?
3. What challenges would learners face if they learn mathematics and its history simultaneously?